Plan of Module 3

1. Knowledge Representation Considerations
   - Extended expressive power
     - ▷ Distinction between strict and defeasible premises
     - ▷ Extending arguments with priorities
     - ▷ Trading sequents by hypersequents
     - ▷ Introducing abducitve sequents
   - Consistency and minimality
   - Alternative formalizations of attack rules
2. **General Properties**
   - **Reasoning with maximal consistency**
   - Rationality postulates

Reasoning with Maximally Consistent Subsets (MCS)

The relation between MCS-based reasoning and argumentation theory has been identified in several works.

Some References:

- O.Arieli, A.Borg, C.Straßer, *Reasoning with maximal consistency by argumentative approaches.* Logic & Computation 28(7):1523–1563, 2018. $\Leftarrow$
- O.Arieli, A.Borg, J.Heyninck, *A review of the relations between logical argumentation and reasoning with maximal consistency.* Annals of Mathematics and Artificial Intelligence 87(3):187–226, 2019.
- L.Amgoud, P.Besnard, *Logical limits of abstract argumentation frameworks*, Applied Non-Classical Logics 23(3):229–267, 2013.
- S.Modgil, H.Prakken, *A general account of argumentation with preferences*, Artificial Intelligence 195:361–397, 2013.
- S.Vesic, *Identifying the class of maxi-consistent operators in argumentation*, Artificial Intelligence Research 47:71–93, 2013.

Maximal Consistent Subsets of Premises

- $\mathfrak{L} = \langle \mathcal{L}, \vdash \rangle$ – a logic, $\mathcal{S}$ – a set of $\mathcal{L}$-formulas.
- $\mathsf{MCS}_{\mathfrak{L}}(\mathcal{S}) = \{\mathcal{T} \mid \mathcal{T}$ is a $\subseteq$-maximally $\vdash$-consistent subsets of $\mathcal{S}\}$.

N. Rescher and R. Manor, *On inference from inconsistent premises*, Theory and Decision 1, pages 179–217, 1970.

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Again, one may define two forms of skeptical entailments as well as a credulous entailment:

- $\mathcal{S} \mid\!\sim^{\cap}_{\mathfrak{L},\mathsf{mcs}} \psi$ if $\psi \in \mathsf{TC}_{\mathfrak{L}}(\bigcap \mathsf{MCS}_{\mathfrak{L}}(\mathcal{S}))$
- $\mathcal{S} \mid\!\sim^{\Cap}_{\mathfrak{L},\mathsf{mcs}} \psi$ if $\psi \in \bigcap_{\mathcal{T} \in \mathsf{MCS}_{\mathfrak{L}}(\mathcal{S})} \mathsf{TC}_{\mathfrak{L}}(\mathcal{T})$
- $\mathcal{S} \mid\!\sim^{\cup}_{\mathfrak{L},\mathsf{mcs}} \psi$ if $\psi \in \bigcup_{\mathcal{T} \in \mathsf{MCS}_{\mathfrak{L}}(\mathcal{S})} \mathsf{TC}_{\mathfrak{L}}(\mathcal{T})$

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Note: $\mathcal{S} \mathrel{|\!\sim}^{\cap}_{\mathfrak{L},\mathsf{mcs}} \psi$ implies $\mathcal{S} \mathrel{|\!\sim}^{\Cap}_{\mathfrak{L},\mathsf{mcs}} \psi$ implies $\mathcal{S} \mathrel{|\!\sim}^{\cup}_{\mathfrak{L},\mathsf{mcs}} \psi$.
The converse is not true (for both implications).

N. Rescher and R. Manor, *On inference from inconsistent premises*, Theory and Decision 1, pages 179–217, 1970.

Some Simple Examples

Example

$$\mathcal{S}_1 = \{p, \neg p, q\}$$

$$\mathrm{MCS}_{\mathrm{CL}}(\mathcal{S}_1) = \{\{p, q\}, \{\neg p, q\}\}$$

- $\mathcal{S}_1 \mid\!\sim^{\star}_{\mathrm{CL,mcs}} q$ but $\mathcal{S}_1 \not\mid\!\sim^{\star}_{\mathrm{CL,mcs}} p$, $\mathcal{S}_1 \not\mid\!\sim^{\star}_{\mathrm{CL,mcs}} \neg p$ $(\star \in \{\cap, \Cap\})$.
- $\mathcal{S}_1 \mid\!\sim^{\cup}_{\mathrm{CL,mcs}} q$, $\mathcal{S}_1 \mid\!\sim^{\cup}_{\mathrm{CL,mcs}} p$, $\mathcal{S}_1 \mid\!\sim^{\cup}_{\mathrm{CL,mcs}} \neg p$ (but $\mathcal{S}_1 \not\mid\!\sim^{\cup}_{\mathrm{CL,mcs}} r$).
- Credulous reasoning does not imply skeptical reasoning.

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- $\mathcal{S}_1 \mathrel{|\!\sim}^{\cup}_{\text{CL,mcs}} q$, $\mathcal{S}_1 \mathrel{|\!\sim}^{\cup}_{\text{CL,mcs}} p$, $\mathcal{S}_1 \mathrel{|\!\sim}^{\cup}_{\text{CL,mcs}} \neg p$ (but $\mathcal{S}_1 \not\mathrel{|\!\sim}^{\cup}_{\text{CL,mcs}} r$).
- Credulous reasoning does not imply skeptical reasoning.

Example

$$\mathcal{S}_2 = \{p \wedge q, \neg p \wedge q\}$$

$$\text{MCS}_{\text{CL}}(\mathcal{S}_2) = \{\{p \wedge q\}, \{\neg p \wedge q\}\}$$

- $\bigcap \text{MCS}(\mathcal{S}_2) = \emptyset$ thus $\mathcal{S}_2 \mathrel{|\!\sim}^{\cap}_{\text{CL,mcs}} \psi$ iff ψ is a CL-tautology.
- Yet, $\mathcal{S}_2 \mathrel{|\!\sim}^{\Cap}_{\text{CL,mcs}} \psi$ when $\psi \in \text{TC}_{\text{CL}}(\{q\})$.
- Different skeptical entailments: $\mathcal{S} \mathrel{|\!\sim}^{\Cap}_{\mathfrak{L},\text{mcs}} \psi \not\Rightarrow \mathcal{S} \mathrel{|\!\sim}^{\cap}_{\mathfrak{L},\text{mcs}} \psi$.

Reminder: MCS-based and AF-based Entailments

$\mathfrak{L} = \langle \mathcal{L}, \vdash \rangle$ – a logic, $\mathcal{S}$ – a set of $\mathcal{L}$-formulas.

- MCS-based entailments
  - $\mathcal{S} \mathrel{|\!\!\sim}^{\cap}_{\mathfrak{L},\text{mcs}} \psi$ if $\psi \in \text{TC}_{\mathfrak{L}}(\bigcap \text{MCS}_{\mathfrak{L}}(\mathcal{S}))$
  - $\mathcal{S} \mathrel{|\!\!\sim}^{\Cap}_{\mathfrak{L},\text{mcs}} \psi$ if $\psi \in \bigcap_{\mathcal{T} \in \text{MCS}_{\mathfrak{L}}(\mathcal{S})} \text{TC}_{\mathfrak{L}}(\mathcal{T})$
  - $\mathcal{S} \mathrel{|\!\!\sim}^{\cup}_{\mathfrak{L},\text{mcs}} \psi$ if $\psi \in \bigcup_{\mathcal{T} \in \text{MCS}_{\mathfrak{L}}(\mathcal{S})} \text{TC}_{\mathfrak{L}}(\mathcal{T})$
- AF-based entailments
  - $\mathcal{S} \mathrel{|\!\!\sim}^{\cap}_{\mathfrak{L},\mathcal{A},\text{sem}} \psi$ if $\exists A \in \bigcap \text{Sem}(\mathcal{AF}(\mathcal{S}))$ with $\text{Conc}(A) = \psi$
  - $\mathcal{S} \mathrel{|\!\!\sim}^{\Cap}_{\mathfrak{L},\mathcal{A},\text{sem}} \psi$ if $\forall \mathcal{E} \in \text{Sem}(\mathcal{AF}(\mathcal{S}))\ \exists A \in \mathcal{E}$ with $\text{Conc}(A) = \psi$
  - $\mathcal{S} \mathrel{|\!\!\sim}^{\cup}_{\mathfrak{L},\mathcal{A},\text{sem}} \psi$ if $\exists A \in \bigcup \text{Sem}(\mathcal{AF}(\mathcal{S}))$ with $\text{Conc}(A) = \psi$

Logical Argumentation and Reasoning with MCSs

Theorem

Let $\mathfrak{L} = \mathsf{CL}$ *and* $\mathcal{A} = \{DirUcut\}$*. Then, for every set of (propositional) formulas* $\mathcal{S}$ *and formula* ψ*,*

- $\mathcal{S} \mid\!\sim^{\cap}_{\mathfrak{L},\mathcal{A},\mathsf{grd}} \psi$ *iff* $\mathcal{S} \mid\!\sim^{\cap}_{\mathfrak{L},\mathcal{A},\mathsf{prf}} \psi$ *iff* $\mathcal{S} \mid\!\sim^{\cap}_{\mathfrak{L},\mathcal{A},\mathsf{stb}} \psi$ *iff* $\mathcal{S} \mid\!\sim^{\cap}_{\mathfrak{L},\mathsf{mcs}} \psi$.
- $\mathcal{S} \mid\!\sim^{\Cap}_{\mathfrak{L},\mathcal{A},\mathsf{prf}} \psi$ *iff* $\mathcal{S} \mid\!\sim^{\Cap}_{\mathfrak{L},\mathcal{A},\mathsf{stb}} \psi$ *iff* $\mathcal{S} \mid\!\sim^{\Cap}_{\mathfrak{L},\mathsf{mcs}} \psi$.
- $\mathcal{S} \mid\!\sim^{\cup}_{\mathfrak{L},\mathcal{A},\mathsf{prf}} \psi$ *iff* $\mathcal{S} \mid\!\sim^{\cup}_{\mathfrak{L},\mathcal{A},\mathsf{stb}} \psi$ *iff* $\mathcal{S} \mid\!\sim^{\cup}_{\mathfrak{L},\mathsf{mcs}} \psi$.

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Theorem

Let $\mathfrak{L} = \mathsf{CL}$ *and* $\emptyset \neq \mathcal{A} \subseteq \{\mathit{Ucut}, \mathit{Def}\}$*. Then:*

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- *If* $\mathcal{S}' = \{\bigvee_i \bigwedge \Gamma_i \mid \Gamma_i$ *is a finite subset of* $\mathcal{S}\}$ *and* $\mathcal{A} = \{\mathit{Ucut}\}$*,* $\mathcal{S}' \mathrel{|\!\sim}^{\Cap}_{\mathfrak{L},\mathcal{A},\mathsf{prf}} \psi$ *iff* $\mathcal{S}' \mathrel{|\!\sim}^{\Cap}_{\mathfrak{L},\mathcal{A},\mathsf{stb}} \psi$ *iff* $\mathcal{S}' \mathrel{|\!\sim}^{\Cap}_{\mathfrak{L},\mathsf{mcs}} \psi$.

Waiving the Maximality Requirement

Example

$$\mathcal{S}_3 = \{p \wedge q, \neg p\}$$

$$\bigcap \mathsf{MCS}_{\mathsf{CL}}(\mathcal{S}_3) = \emptyset, \quad \bigcap_{\mathcal{T} \in \mathsf{MCS}_{\mathsf{CL}}(\mathcal{S}_3)} \mathsf{TC}_{\mathsf{CL}}(\mathcal{T}) = \emptyset$$

- Thus, $\mathcal{S}_3 \mid\!\sim^{\cap}_{\mathsf{CL},\mathsf{mcs}} \psi$ and $\mathcal{S}_3 \mid\!\sim^{\Cap}_{\mathsf{CL},\mathsf{mcs}} \psi$ iff ψ a tautology

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Definition (Benferhat, Dubois & Prade, 1997)

$\mathcal{S} \mid\!\sim_{\mathfrak{L},\mathsf{cs}} \psi$ iff

- there is some $\vdash_{\mathfrak{L}}$-consistent $\mathcal{T} \subseteq \mathcal{S}$ such that $\mathcal{T} \vdash_{\mathfrak{L}} \psi$, and
- there is no $\vdash_{\mathfrak{L}}$-consistent $\mathcal{T}' \subseteq \mathcal{S}$ such that $\mathcal{T}' \vdash_{\mathfrak{L}} \neg\psi$.

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Note: If $\mathcal{S} \mid\!\sim^{\cap}_{\mathfrak{L},\mathsf{mcs}} \psi$ then $\mathcal{S} \mid\!\sim_{\mathfrak{L},\mathsf{cs}} \psi$.
The converse is not true: $\mathcal{S}_3 \mid\!\sim_{\mathsf{CL,cs}} q$ but $\mathcal{S}_3 \not\mid\!\sim^{\cap}_{\mathsf{CL,mcs}} q$.

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Characterization of $\mid\!\sim_{cs}$ by Dung's Semantics

Recall:

Consistency Undercut (ConUcut)

$$\frac{\Rightarrow \neg \bigwedge \Gamma_2' \qquad \Gamma_2, \Gamma_2' \Rightarrow \psi_2}{\Gamma_2, \Gamma_2' \not\Rightarrow \psi_2}$$

(An argument with inconsistent premises is attacked and cannot be defended)

Defeating Rebuttal (DefReb)

$$\frac{\Gamma_1 \Rightarrow \psi_1 \qquad \psi_1 \Rightarrow \neg \psi_2 \qquad \Gamma_2 \Rightarrow \psi_2}{\Gamma_2 \not\Rightarrow \psi_2}$$

(Conflicting conclusions)

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(Conflicting conclusions)

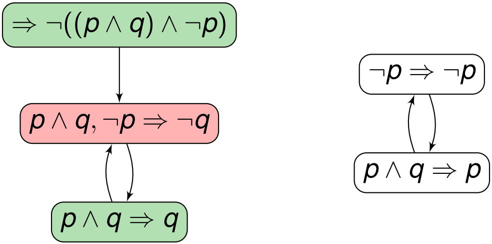
Theorem

Let $\mathfrak{L} = \text{CL}$ *and* $\mathcal{A} = \{$*DefReb, ConUcut*$\}$. *Then:*

$\mathcal{S} \mid\!\sim^{\cap}_{\mathfrak{L},\mathcal{A},\text{grd}} \psi$ *iff* $\mathcal{S} \mid\!\sim^{\cap}_{\mathfrak{L},\mathcal{A},\text{prf}} \psi$ *iff* $\mathcal{S} \mid\!\sim^{\cap}_{\mathfrak{L},\mathcal{A},\text{stb}} \psi$ *iff* $\mathcal{S} \mid\!\sim_{\mathfrak{L},\text{cs}} \psi$.

Example

Part of the argumentation framework for $\mathcal{S}_3 = \{p \wedge q, \neg p\}$,
based on classical logic and the attack rules $\mathcal{A} = \{\text{ConUcut}, \text{DefReb}\}$.



$\mathcal{S}_3 \mid\!\sim^{\cap}_{\mathrm{CL},\mathcal{A},\mathrm{grd}} q$ $\quad\mathcal{S}_3 \mid\!\sim^{\cap}_{\mathrm{CL},\mathcal{A},\mathrm{prf}} q$ $\quad\mathcal{S}_3 \mid\!\sim^{\cap}_{\mathrm{CL},\mathcal{A},\mathrm{stb}} q$ $\quad\mathcal{S}_2 \mid\!\sim_{\mathrm{CL},\mathrm{cs}} q$

$\mathcal{S}_3 \mid\!\not\sim^{\cap}_{\mathrm{CL},\mathcal{A},\mathrm{grd}} p$ $\quad\mathcal{S}_3 \mid\!\not\sim^{\cap}_{\mathrm{CL},\mathcal{A},\mathrm{prf}} p$ $\quad\mathcal{S}_3 \mid\!\not\sim^{\cap}_{\mathrm{CL},\mathcal{A},\mathrm{stb}} p$ $\quad\mathcal{S}_3 \mid\!\not\sim_{\mathrm{CL},\mathrm{cs}} p.$

$\mathcal{S}_3 \mid\!\not\sim^{\cap}_{\mathrm{CL},\mathcal{A},\mathrm{grd}} \neg p$ $\quad\mathcal{S}_3 \mid\!\not\sim^{\cap}_{\mathrm{CL},\mathcal{A},\mathrm{prf}} \neg p$ $\quad\mathcal{S}_3 \mid\!\not\sim^{\cap}_{\mathrm{CL},\mathcal{A},\mathrm{stb}} \neg p$ $\quad\mathcal{S}_3 \mid\!\not\sim_{\mathrm{CL},\mathrm{cs}} \neg p.$

Dung's Semantics and MCS-based Reasoning: Some Counter-Examples

1. $\mathfrak{L} = \mathsf{CL}$, $\mathcal{A} = \{\mathsf{Ucut}\}$, type $= \cap$, sem $=$ naive ($\subseteq$-maximal c.f. sets).

Note: $\mathcal{E} = \{p \wedge \neg p \ \Rightarrow \psi \mid \psi$ is *not* a CL-tautology$\}$ is a naive extension of the AF for $\mathcal{S} = \{p \wedge \neg p\}$.

Proof: *Conflict-freeness*: $\Gamma \Rightarrow \psi$ Ucut-attacks $\epsilon \in \mathcal{E}$ iff ψ is logically equivalent to $\neg(p \wedge \neg p)$, i.e., ψ is a CL-tautology. Thus $\Gamma \Rightarrow \psi \notin \mathcal{E}$.
Maximality: the only arguments that are excluded from $\mathcal{E}$ are those whose conclusion is a CL-tautology. $\square$

Thus: $\mathcal{S} \mid\!\sim^{\cap}_{\mathsf{CL,mcs}} \neg(p \wedge \neg p)$, but $\mathcal{S} \not\mid\!\sim^{\cap}_{\mathsf{CL,Ucut,naive}} \neg(p \wedge \neg p)$.

Dung's Semantics and MCS-based Reasoning: Some Counter-Examples

2. $\mathfrak{L} = \mathsf{CL}$, $\mathcal{A} = \{\mathsf{Ucut}\}$, type $= \Cap$, sem $=$ stb.

<u>Note</u>: $\mathcal{E} = \mathsf{Arg}_{\mathsf{CL}}(\{p \wedge r_1\}) \cup \mathsf{Arg}_{\mathsf{CL}}(\{q \wedge r_2\}) \cup \mathsf{Arg}_{\mathsf{CL}}(\{\neg(p \wedge q) \wedge r_3\})$ is a stable extension of the AF for $\mathcal{S} = \{p \wedge r_1,\ q \wedge r_2,\ \neg(p \wedge q) \wedge r_3\}$.

<u>Proof</u>: Follows from the characterization of the stable extensions in this case (see is what follows). $\square$

<u>Thus</u>: $\mathcal{S} \mathrel{|\!\sim}^{\Cap}_{\mathsf{CL,mcs}} (r_1 \wedge r_2) \vee (r_1 \wedge r_3) \vee (r_2 \wedge r_3)$, but
$\mathcal{S} \not\mathrel{|\!\sim}^{\Cap}_{\mathsf{CL,Ucut,stb}} (r_1 \wedge r_2) \vee (r_1 \wedge r_3) \vee (r_2 \wedge r_3)$.

(There is no argument of the form $\Gamma \Rightarrow (r_1 \wedge r_2) \vee (r_1 \wedge r_3) \vee (r_2 \wedge r_3) \in \mathcal{E}$ for which $\Gamma \subseteq \mathcal{S}$).

More General Characterizations

The results so far are extended from CL to any logic, in which at least the following basic rules are admissible:

$$[\text{Ref}]\ \frac{}{\phi \Rightarrow \phi} \qquad [\text{Cut}]\ \frac{\Gamma_1 \Rightarrow \psi, \Pi_1 \quad \Gamma_2, \psi \Rightarrow \Delta_2}{\Gamma_1, \Gamma_2 \Rightarrow \Pi_1, \Delta_2}$$

$$[\text{LMon}]\ \frac{\Gamma \Rightarrow \Delta}{\Gamma, \phi \Rightarrow \Delta} \qquad [\text{RMon}]\ \frac{\Gamma \Rightarrow \Pi}{\Gamma \Rightarrow \Pi, \phi}$$

$$[\neg\Rightarrow]\ \frac{\Gamma \Rightarrow \Pi, \varphi}{\neg\varphi, \Gamma \Rightarrow \Pi} \qquad [\Rightarrow\neg]\ \frac{\varphi, \Gamma \Rightarrow \Pi}{\Gamma \Rightarrow \Pi, \neg\varphi}$$

$$[\wedge\Rightarrow]\ \frac{\Gamma, \varphi, \psi \Rightarrow \Delta}{\Gamma, \varphi \wedge \psi \Rightarrow \Delta} \qquad [\Rightarrow\wedge]\ \frac{\Gamma_1 \Rightarrow \Pi_1, \varphi \quad \Gamma_2 \Rightarrow \Pi_2, \psi}{\Gamma_1, \Gamma_2 \Rightarrow \Pi_1, \Pi_2, \varphi \wedge \psi}$$

(in single-conclusion calculi Π, Π_1, Π_2 are empty, and Δ, Δ_2 contain at most one formula).

More General Characterizations

The attack rules $\mathcal{A}$ in $\{\text{Def}, \text{DirDef}, \text{Ucut}, \text{DirUcut}, \text{ConUcut}\}$ are divided to three types:

- sub: At least one attack is Undercut or Defeat (i.e., $\mathcal{A} \cap \{\text{Def}, \text{Ucut}\} \neq \emptyset$), thus an argument can be attacked on a subset of its support,
- dir: A non-empty set of direct attack rules (i.e., $\emptyset \neq \mathcal{A} \subseteq \{\text{DDef}, \text{DUcut}\}$),
- con: A non-empty set of direct attack rules and ConUcut (i.e., $\{\text{ConUcut}\} \subsetneq \mathcal{A} \subseteq \{\text{ConUcut}, \text{DDef}, \text{DUcut}\}$).

Characterizations of Extensions by Consistent Sets

Theorem (Arieli, Borg, Straßer (KR'2021))

Given an argumentation framework $\mathcal{AF}(\mathcal{S}) = \langle \text{Arg}_{\mathfrak{L}}(\mathcal{S}), \text{Attack}(\mathcal{A}) \rangle$, *where the rules in* $\text{Attack}(\mathcal{A})$ *are of type* $\text{AT} \in \{\text{dir}, \text{con}, \text{set}\}$. *Then:*

- $\text{Sem}(\mathcal{AF}(\mathcal{S})) = \{\text{Arg}_{\mathfrak{L}}(\mathcal{T}) \mid \mathcal{T} \in \text{MCS}_{\mathfrak{L}}(\mathcal{S})\}$ *where* $\text{Sem} \in \{\text{Prf}, \text{Stb}, \text{SStb}\}$ *and* $\text{AT} \in \{\text{dir}, \text{con}\}$.
- $\text{Sem}(\mathcal{AF}(\mathcal{S})) = \{\text{Arg}_{\mathfrak{L}}(\omega) \mid \omega \in \Omega_{\mathfrak{L}}(\mathcal{S})\}$ *where* $\text{Sem} \in \{\text{Prf}, \text{Stb}, \text{SStb}\}$ *and* $\text{AT} = \text{set}$.
- $\text{Sem}(\mathcal{AF}(\mathcal{S})) = \{\text{Arg}_{\mathfrak{L}}(\bigcap \text{MCS}_{\mathfrak{L}}(\mathcal{S}))\}$ *where* $\text{Sem} = \text{Grd}$ *and* $\text{AT} \in \{\text{set}, \text{con}\}$.
- $\text{Sem}(\mathcal{AF}(\mathcal{S})) = \{\text{Arg}_{\mathfrak{L}}(\mathcal{T})\}$ *where* $\text{Sem} = \text{Grd}$ *and* $\text{AT} = \text{dir}$. *Here:* $\mathcal{T} = \{\psi \in \mathcal{S} \mid \psi$ *is* $\vdash$*-consistent*$\}$ *if this set is* $\vdash$*-consistent and* $\mathcal{T} = \emptyset$ *otherwise.*

$\Omega_{\mathfrak{L}}(\mathcal{S})$ is the set of subsets of $2^{\mathcal{S}}$, where for every $\omega \in \Omega_{\mathfrak{L}}(\mathcal{S})$ it holds that:

a) the elements of ω are pairwise $\vdash_{\mathcal{X}}$-consistent: $\mathcal{T}_i \cup \mathcal{T}_j$ is $\vdash_{\mathcal{X}}$-consistent for every $\mathcal{T}_i, \mathcal{T}_j \in \omega$.

b) for every finite set $\Theta \in 2^{\mathcal{S}}$ there is a set $\mathcal{T} \in \omega$ such that either $\Theta \subseteq \mathcal{T}$ or $\Theta \cup \mathcal{T}$ is $\vdash$-inconsistent.

For $\omega \in \Omega^{\mathcal{X}}_{\mathfrak{L}}(\mathcal{S})$, we let $\text{Arg}_{\mathfrak{L}}(\omega) = \bigcup_{\mathcal{T} \in \omega} \text{Arg}_{\mathfrak{L}}(\mathcal{T})$.

Example (for $\mathfrak{L} = \mathsf{CL}$)

$\mathcal{S}$	Semantics	Attack Type	Extensions
$\{p\}$	All	All	$\mathsf{Arg}_{\mathsf{CL}}(\{p\})$

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$\mathcal{S}$	Semantics	Attack Type	Extensions
$\{p\}$	All	All	$\mathsf{Arg}_{\mathsf{CL}}(\{p\})$
$\{p, \neg p\}$	Grd	All	$\mathsf{Arg}_{\mathsf{CL}}(\emptyset)$
$\{p, \neg p\}$	Prf, Stb, SStb	All	$\mathsf{Arg}_{\mathsf{CL}}(\{p\})$, $\mathsf{Arg}_{\mathsf{CL}}(\{\neg p\})$

Example (for $\mathfrak{L} = \mathsf{CL}$)

$\mathcal{S}$	Semantics	Attack Type	Extensions
$\{p\}$	All	All	$\mathsf{Arg}_{\mathsf{CL}}(\{p\})$
$\{p, \neg p\}$	Grd	All	$\mathsf{Arg}_{\mathsf{CL}}(\emptyset)$
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$\{p, \neg p, q\}$	Grd	All	$\mathsf{Arg}_{\mathsf{CL}}(\{q\})$
$\{p, \neg p, q\}$	Prf, Stb, SStb	All	$\mathsf{Arg}_{\mathsf{CL}}(\{p, q\})$, $\mathsf{Arg}_{\mathsf{CL}}(\{\neg p, q\})$

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$\mathcal{S}$	Semantics	Attack Type	Extensions
$\{p\}$	All	All	$\text{Arg}_{\text{CL}}(\{p\})$
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$\{p, \neg p\}$	Prf, Stb, SStb	All	$\text{Arg}_{\text{CL}}(\{p\})$, $\text{Arg}_{\text{CL}}(\{\neg p\})$
$\{p, \neg p, q\}$	Grd	All	$\text{Arg}_{\text{CL}}(\{q\})$
$\{p, \neg p, q\}$	Prf, Stb, SStb	All	$\text{Arg}_{\text{CL}}(\{p, q\})$, $\text{Arg}_{\text{CL}}(\{\neg p, q\})$
$\{\psi_1, \psi_2, \psi_3\}$	Prf, Stb, SStb	dir, con	$\mathcal{E}_1,\ \mathcal{E}_2,\ \mathcal{E}_3$
$\{\psi_1, \psi_2, \psi_3\}$	Prf, Stb, SStb	set	$\mathcal{E}_1,\ \mathcal{E}_2,\ \mathcal{E}_3,\ \mathcal{E}_4$

$\psi_1 = p \wedge r,\ \psi_2 = q \wedge r,\ \psi_3 = \neg(p \wedge q)$.

$\mathcal{E}_1 = \text{Arg}_{\text{CL}}(\{\psi_1, \psi_2\}),\ \mathcal{E}_2 = \text{Arg}_{\text{CL}}(\{\psi_2, \psi_3\}),\ \mathcal{E}_3 = \text{Arg}_{\text{CL}}(\{\psi_1, \psi_3\}),$

$\mathcal{E}_4 = \text{Arg}_{\text{CL}}(\{\psi_1\}) \cup \text{Arg}_{\text{CL}}(\{\psi_2\}) \cup \text{Arg}_{\text{CL}}(\{\psi_3\})$.

Characterizations of AF-based Entailments

Theorem

Given an argumentation framework $\mathcal{AF}(\mathcal{S}) = \langle \mathsf{Arg}_{\mathfrak{L}}(\mathcal{S}), \mathsf{Attack}(\mathcal{A})\rangle$, *where the rules in* $\mathsf{Attack}(\mathcal{A})$ *are of type* $\mathsf{AT} \in \{\mathsf{dir}, \mathsf{con}, \mathsf{set}\}$. *Then:*

- $\mathcal{S} \mid\!\sim^{\cap}_{\mathfrak{L},\mathsf{AT},\mathsf{sem}} \psi$ *iff* $\mathcal{S} \mid\!\sim^{\cap}_{\mathfrak{L},\mathsf{mcs}} \psi$ *for every* $\mathsf{Sem} \in \{\mathsf{Prf}, \mathsf{Stb}, \mathsf{SStb}\}$ *and* $\mathsf{AT} \in \{\mathsf{con}, \mathsf{dir}\}$.
- $\mathcal{S} \mid\!\sim^{\cap}_{\mathfrak{L},\mathsf{AT},\mathsf{sem}} \psi$ *iff* $\mathcal{S} \mid\!\sim^{\cap}_{\mathfrak{L},\mathsf{mcs}} \psi$ *for every* $\mathsf{Sem} = \mathsf{Grd}$ *and* $\mathsf{AT} \in \{\mathsf{con}, \mathsf{set}\}$.
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- $\mathcal{S} \mid\!\sim^{\cup}_{\mathfrak{L},\mathsf{AT},\mathsf{sem}} \psi$ *iff* $\mathcal{S} \mid\!\sim^{\cup}_{\mathfrak{L},\mathsf{mcs}} \psi$ *for every* $\mathsf{Sem} \in \{\mathsf{Prf}, \mathsf{Stb}, \mathsf{SStb}\}$ *and* $\mathsf{AT} \in \{\mathsf{con}, \mathsf{set}, \mathsf{dir}\}$.
- $\mathcal{S} \mid\!\sim^{\star}_{\mathfrak{L},\mathsf{AT},\mathsf{sem}} \psi$ *iff* $\mathcal{S} \mid\!\sim^{\star}_{\mathfrak{L},\Omega} \psi$ *for every* $\star \in \{\cap, \Cap, \cup\}$, $\mathsf{Sem} \in \{\mathsf{Prf}, \mathsf{Stb}, \mathsf{SStb}\}$, *and* $\mathsf{AT} = \mathsf{set}$.

Plan of Module 3

1. Knowledge Representation Considerations
    - Extended expressive power
        - ▷ Distinction between strict and defeasible premises
        - ▷ Extending arguments with priorities
        - ▷ Trading sequents by hypersequents
        - ▷ Introducing abducitve sequents
    - Consistency and minimality
    - Alternative formalizations of attack rules
2. General Properties
    - Reasoning with maximal consistency
    - **Rationality postulates**

Motivation

A logical argumentation framework is affected by a variety of factors:

- The language of the arguments
- The underlying logic
- The type of the attacks
- Which extensions are considered (the semantics)
- The type of aggregation for reasoning (credulous/skeptical)

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A postulates-based method:

Construction of AFs in terms of some list of desiderata
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- Properties of individual extensions (for credulous reasoning)
- Properties of the whole Sem-extensions (for skeptical reasoning)
- Properties of the induced entailment relations (NMR-related & paraconsistency-related postulates)

Motivation (Cont'd.)

$$\mathfrak{L} = \mathsf{CL}, \;\; \mathcal{R} = \{\mathrm{DirDef}, \mathrm{ConUcut}\}, \;\; \mathcal{S} = \{p, q, \neg p \vee \neg q, r\}$$

$a_1 = r \Rightarrow r$ $\quad a_4 = \neg p \vee \neg q \Rightarrow \neg p \vee \neg q$ $\quad a_7 = p, q \Rightarrow p \wedge q$

$a_2 = p \Rightarrow p$ $\quad a_5 = p \Rightarrow \neg((\neg p \vee \neg q) \wedge q)$ $\quad a_8 = \neg p \vee \neg q, q \Rightarrow \neg p$

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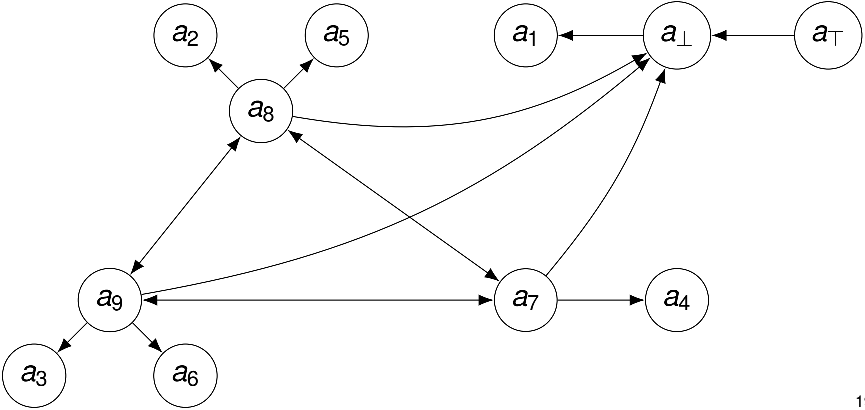
$a_\top = \Rightarrow \neg(p \wedge q \wedge (\neg p \vee \neg q))$ $\quad a_\bot = p, q, \neg p \vee \neg q \Rightarrow \neg r$



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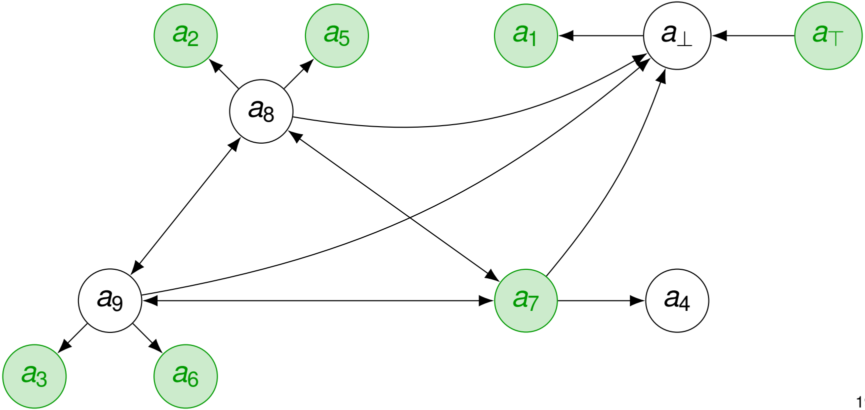

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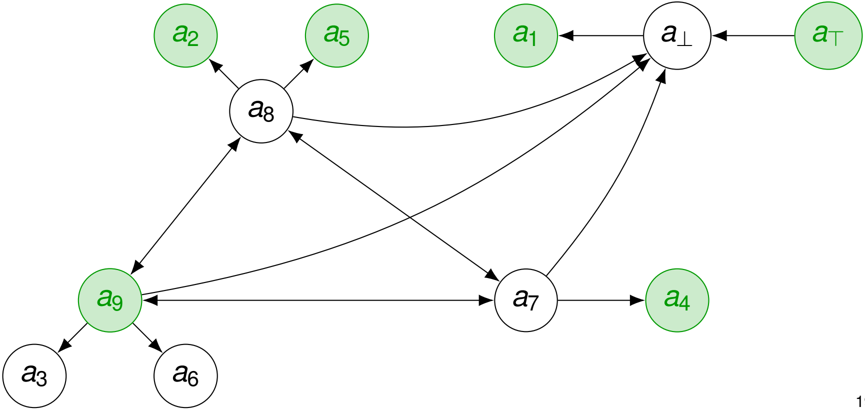

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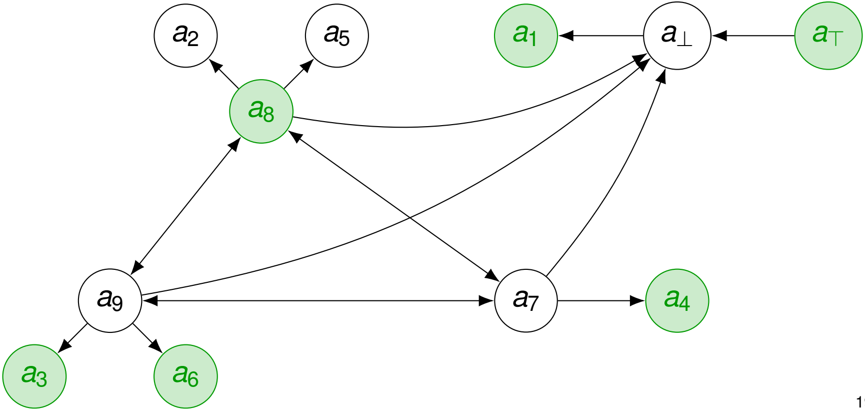

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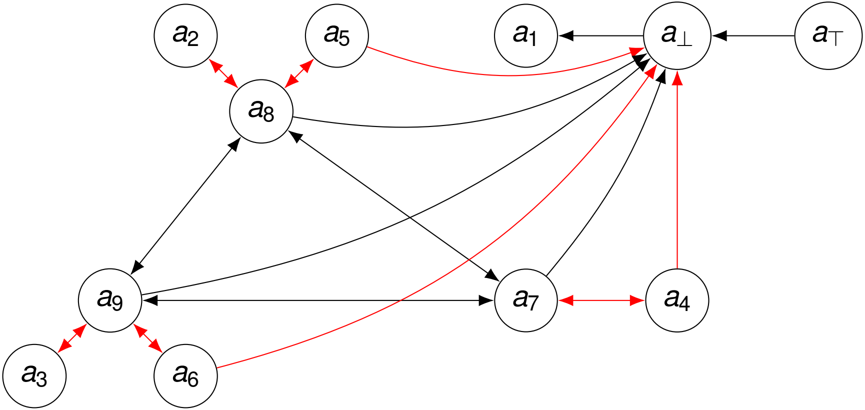

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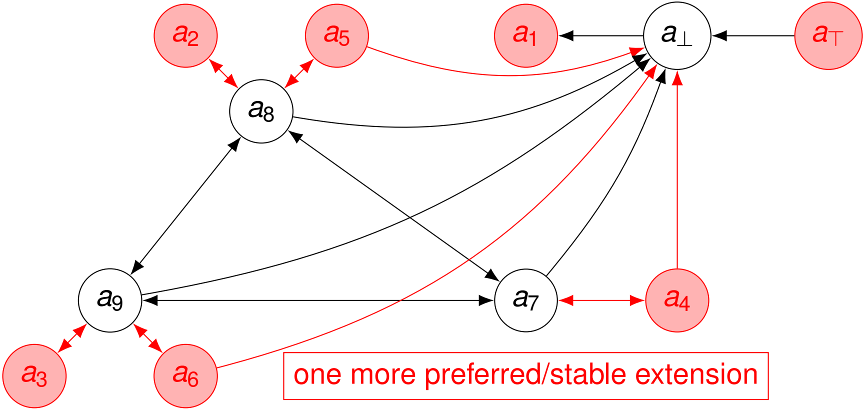

one more preferred/stable extension

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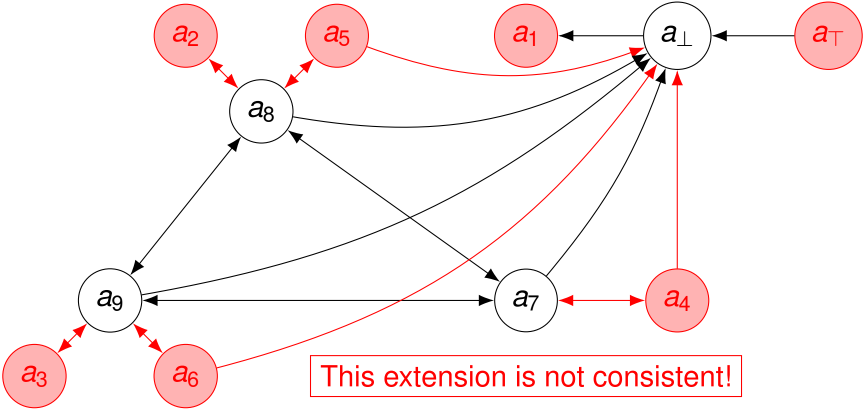

Rationality Postulates (For Individual Extensions)

Given: $\mathcal{AF}(\mathcal{S}) = \langle \text{Arg}_{\mathfrak{L}}(\mathcal{S}), \text{Attack} \rangle$, $\text{Sem} \in \{\text{cmp}, \text{grd}, \text{prf}, \text{sstb}\}$.

Let: $\mathcal{E} \in \text{Sem}(\mathcal{AF}_{\mathfrak{L}}(\mathcal{S}))$, $A \in \text{Arg}_{\mathfrak{L}}(\mathcal{S})$.

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- Closure postulates:
  - *closure of extensions:* $\text{TC}_{\mathfrak{L}}(\text{Concs}(\mathcal{E})) = \text{Concs}(\mathcal{E})$.
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  - *closure of extensions:* $\text{TC}_{\mathfrak{L}}(\text{Concs}(\mathcal{E})) = \text{Concs}(\mathcal{E})$.
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M.Caminada and L.Amgoud, *On the evaluation of argumentation formalisms*, Artificial Intelligence 171(5):286–310, 2007.

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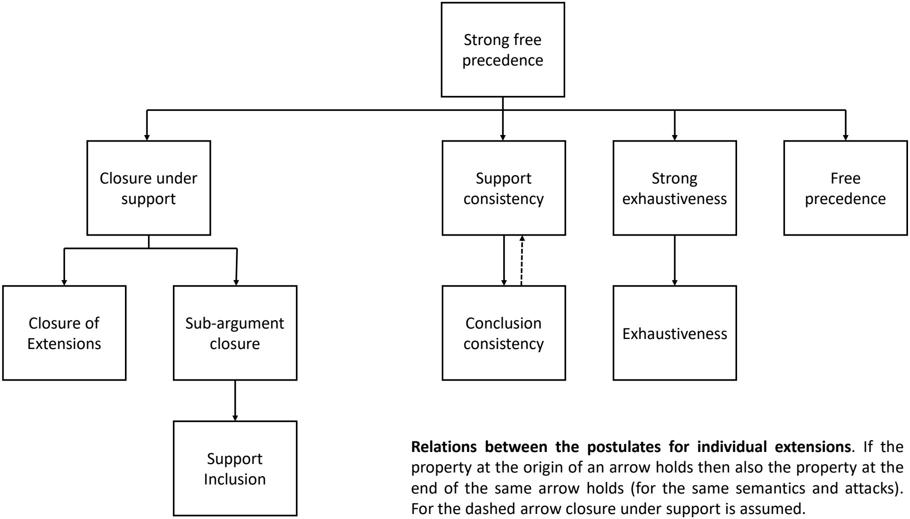
Relations Among The Postulates



Relations between the postulates for individual extensions. If the property at the origin of an arrow holds then also the property at the end of the same arrow holds (for the same semantics and attacks). For the dashed arrow closure under support is assumed.

Reminder: The General Setting

1. Thee logic has a S&C calculus with the following admissible rules:

[Ref] $$\frac{}{\phi \Rightarrow \phi}$$

[Cut] $$\frac{\Gamma_1 \Rightarrow \psi, \Pi_1 \quad \Gamma_2, \psi \Rightarrow \Delta_2}{\Gamma_1, \Gamma_2 \Rightarrow \Pi_1, \Delta_2}$$

[LMon] $$\frac{\Gamma \Rightarrow \Delta}{\Gamma, \phi \Rightarrow \Delta}$$

[RMon] $$\frac{\Gamma \Rightarrow \Pi}{\Gamma \Rightarrow \Pi, \phi}$$

[$\neg\Rightarrow$] $$\frac{\Gamma \Rightarrow \Pi, \varphi}{\neg\varphi, \Gamma \Rightarrow \Pi}$$

[$\Rightarrow\neg$] $$\frac{\varphi, \Gamma \Rightarrow \Pi}{\Gamma \Rightarrow \Pi, \neg\varphi}$$

[$\wedge\Rightarrow$] $$\frac{\Gamma, \varphi, \psi \Rightarrow \Delta}{\Gamma, \varphi \wedge \psi \Rightarrow \Delta}$$

[$\Rightarrow\wedge$] $$\frac{\Gamma_1 \Rightarrow \Pi_1, \varphi \quad \Gamma_2 \Rightarrow \Pi_2, \psi}{\Gamma_1, \Gamma_2 \Rightarrow \Pi_1, \Pi_2, \varphi \wedge \psi}$$

2. The attack rules $\mathcal{A}$ are divided to three types:

sub $\mathcal{A} \cap \{\text{Def}, \text{Ucut}\} \neq \emptyset$

dir $\emptyset \neq \mathcal{A} \subseteq \{\text{DDef}, \text{DUcut}\}$

con $\{\text{ConUcut}\} \subsetneq \mathcal{A} \subseteq \{\text{ConUcut}, \text{DDef}, \text{DUcut}\}$

Postulate-Based Study – Summary of the Results

	dir-attacks	con-attacks	sub-attacks
closure of extensions	✓	✓	grd
closure under support	✓	✓	grd
sub-argument closure	✓	✓	✓
support inclusion	✓	✓	✓
consistency	✓	✓	grd
support consistency	✓	✓	grd
free precedence	grd	✓	✓
strong fee precedence	–	grd	grd
exhaustiveness	prf, stb, sstb	✓	grd
strong exhaustiveness	prf, stb, sstb	✓	grd

(✓ = cmp, grd, prf, stb, sstb)

O. Arieli, A. Borg and C. Straßer, *Tuning logical argumentation frameworks: A postulate-derived approach*, FLAIRS-33 (UR Track), pages 557–562, AAAI Press, 2020.

Rationality Postulates (For Sets of Extensions)

- *maximal consistency*: $\mathsf{Sem}(\mathcal{AF}(\mathcal{S})) = \{\mathsf{Arg}_{\mathfrak{L}}(\mathcal{T}) \mid \mathcal{T} \in \mathsf{MCS}_{\mathfrak{L}}(\mathcal{S})\}$
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	dir-attacks	con-attacks	sub-attacks
maximal consistency	prf, stb, sstb	prf, stb, sstb	–
weak maximal consistency	prf, stb, sstb	prf, stb, sstb	prf, stb, sstb
stability	✓	✓	✓
strong stability	prf, stb, sstb	prf, stb, sstb	prf, stb, sstb

Rationality Postulates (For the Induced Entailments)

Rationality Postulates (For the Induced Entailments)

1. General Patterns of NMR (The KLM Postulates)

Given a logic $\mathfrak{L} = \langle \mathcal{L}, \vdash \rangle$, a relation $\mid\!\sim$ on $2^{\mathsf{WFF}(\mathcal{L})} \times \mathsf{WFF}(\mathcal{L})$ is:

- $\vdash$-cumulative, if it satisfies:
  - $\vdash$-cautious reflexivity: $\phi \mid\!\sim \phi$ for a $\vdash$-consistent ϕ.
  - $\vdash$-right weakening: $\mathcal{S} \mid\!\sim \phi$ and $\phi \vdash \psi$ imply $\mathcal{S} \mid\!\sim \psi$.
  - $\vdash$-left logical equivalence: If $\mathcal{S}, \phi \mid\!\sim \sigma$, $\psi \vdash \phi$, $\phi \vdash \psi$, then $\mathcal{S}, \psi \mid\!\sim \sigma$.
  - cautious monotonicity: if $\mathcal{S} \mid\!\sim \phi$ and $\mathcal{S} \mid\!\sim \psi$, then $\mathcal{S}, \phi \mid\!\sim \psi$.
  - cautious cut: if $\mathcal{S} \mid\!\sim \psi$ and $\mathcal{S}, \psi \mid\!\sim \phi$, then $\mathcal{S} \mid\!\sim \phi$.
- $\vdash$-preferential, if it is $\vdash$-cumulative and satisfies:
  - or: if $\mathcal{S}, \phi \mid\!\sim \sigma$ and $\mathcal{S}, \psi \mid\!\sim \sigma$, then $\mathcal{S}, \phi \vee \psi \mid\!\sim \sigma$.
- $\vdash$-rational, if it is $\vdash$-preferential and satisfies:
  - rational monotonicity: if $\mathcal{S} \mid\!\sim \psi$ and $\mathcal{S} \not\mid\!\sim \neg\phi$, then $\mathcal{S}, \phi \mid\!\sim \psi$.

S. Kraus, D. Lehmann and M. Magidor, *Nonmonotonic reasoning, preferential models and cumulative logics*, Artificial Intelligence 44(1–2), pages 167–207, 1990.

General Patterns of NMR – Summary of the Results

	$\mid\!\sim^{\cap}_{\mathfrak{L},\mathsf{AT},\mathsf{sem}}$	$\mid\!\sim^{\Cap}_{\mathfrak{L},\mathsf{AT},\mathsf{sem}}$	$\mid\!\sim^{\cup}_{\mathfrak{L},\mathsf{AT},\mathsf{sem}}$
cumulativity, AT $\in$ {con, dir}	✓	✓	grd
cumulativity, AT $=$ set	✓	grd	grd
preferentiality, AT $=$ con	–	prf, stb, sstb	–
preferentiality, AT $=$ dir	grd	✓	grd
preferentiality, AT $=$ set	–	–	–
rationality, AT $\in$ {set, con}	–	–	–
rationality, AT $=$ dir	grd	grd	grd
monoton., AT $\in$ {dir, con, set}	–	–	prf, stb, sstb

(✓ = grd, prf, stb, sstb)

O. Arieli, A. Borg and C. Straßer, *Characterizations and classifications of argumentative entailments*, KR'21, 2021.

Some Further Details: Monotonicity

- For every sem $\in \{\mathsf{prf}, \mathsf{stb}, \mathsf{sstb}\}$ and AT $\in \{\mathsf{dir}, \mathsf{con}, \mathsf{set}\}$, $\mid\!\sim^{\cup}_{\mathfrak{L},\mathsf{AT},\mathsf{sem}}$ is monotonic.

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Proof: It is sufficient to show that $\mid\!\sim^{\cup}_{\mathfrak{L},\text{mcs}}$ is monotonic. Indeed,

$$\begin{aligned}
\mathcal{S} \mid\!\sim^{\cup}_{\mathfrak{L},\text{mcs}} \psi \quad & \Rightarrow \exists \mathcal{T} \in \text{MCS}_{\mathfrak{L}}(\mathcal{S}) \text{ s.t. } \mathcal{T} \vdash \psi \\
& \Rightarrow \mathcal{T} \in \text{CS}_{\mathfrak{L}}(\mathcal{S} \cup \mathcal{S}') \\
& \Rightarrow \exists \mathcal{T}' \in \text{MCS}_{\mathfrak{L}}(\mathcal{S} \cup \mathcal{S}') \text{ s.t. } \mathcal{T} \subseteq \mathcal{T}' \\
& \Rightarrow \exists \mathcal{T}' \in \text{MCS}_{\mathfrak{L}}(\mathcal{S} \cup \mathcal{S}') \text{ s.t. } \mathcal{T}' \vdash \psi \quad \text{(since } \vdash \text{ is monotonic)} \\
& \Rightarrow \mathcal{S}, \mathcal{S}' \mid\!\sim^{\cup}_{\mathfrak{L},\text{mcs}} \psi \qquad \square
\end{aligned}$$

Some Further Details: Monotonicity

- For every sem $\in \{\mathsf{prf}, \mathsf{stb}, \mathsf{sstb}\}$ and AT $\in \{\mathsf{dir}, \mathsf{con}, \mathsf{set}\}$, $\mid\!\sim^{\cup}_{\mathfrak{L},\mathsf{AT},\mathsf{sem}}$ is monotonic.

  Proof: It is sufficient to show that $\mid\!\sim^{\cup}_{\mathfrak{L},\mathsf{mcs}}$ is monotonic. Indeed,

$$\begin{aligned} \mathcal{S} \mid\!\sim^{\cup}_{\mathfrak{L},\mathsf{mcs}} \psi \quad & \Rightarrow \exists \mathcal{T} \in \mathsf{MCS}_{\mathfrak{L}}(\mathcal{S}) \text{ s.t. } \mathcal{T} \vdash \psi \\ & \Rightarrow \mathcal{T} \in \mathsf{CS}_{\mathfrak{L}}(\mathcal{S} \cup \mathcal{S}') \\ & \Rightarrow \exists \mathcal{T}' \in \mathsf{MCS}_{\mathfrak{L}}(\mathcal{S} \cup \mathcal{S}') \text{ s.t. } \mathcal{T} \subseteq \mathcal{T}' \\ & \Rightarrow \exists \mathcal{T}' \in \mathsf{MCS}_{\mathfrak{L}}(\mathcal{S} \cup \mathcal{S}') \text{ s.t. } \mathcal{T}' \vdash \psi \quad \text{(since } \vdash \text{ is monotonic)} \\ & \Rightarrow \mathcal{S}, \mathcal{S}' \mid\!\sim^{\cup}_{\mathfrak{L},\mathsf{mcs}} \psi \qquad \square \end{aligned}$$

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- Monotonicity ceases to hold for grounded semantics and for skeptical reasoning ($\cap, \Cap$)

  Example: Let $\mathfrak{L} = \mathsf{CL}$, $\mathcal{S} = \{p\}$, $\mathcal{S}' = \{\neg p\}$.

  $\mathsf{Sem}(\mathcal{AF}(\{p\})) = \{\mathsf{Arg}_{\mathsf{CL}}(p)\}$, $\mathsf{Grd}(\mathcal{AF}(\{p, \neg p\})) = \{\mathsf{Arg}_{\mathsf{CL}}(\emptyset)\}$,
  $\mathsf{Sem}(\mathcal{AF}(\{p, \neg p\})) = \{\mathsf{Arg}_{\mathsf{CL}}(p), \mathsf{Arg}_{\mathsf{CL}}(\neg p)\}$ ($\mathsf{Sem} \in \{\mathsf{Prf}, \mathsf{Stb}, \mathsf{SStb}\}$).

  - $p \mid\!\sim^{\cap}_{\mathsf{CL},\mathsf{AT},\mathsf{grd}} p$ but $p, \neg p \not\mid\!\sim^{\cap}_{\mathsf{CL},\mathsf{AT},\mathsf{grd}} p$
  - $p \mid\!\sim^{\cap}_{\mathsf{CL},\mathsf{AT},\mathsf{sem}} p$ but $p, \neg p \not\mid\!\sim^{\cap}_{\mathsf{CL},\mathsf{AT},\mathsf{sem}} p$ (Similarly for $\Cap$).

Some Further Details: Cumulativity

- Positive results: By checking the properties w.r.t. the corresponding consistency-based entailments.
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- $\mid\!\sim^{\cap}_{\mathfrak{L},\text{mcs}}$ satisfies cautious cut:

  $\mathcal{S}, \psi \mid\!\sim^{\cap}_{\mathfrak{L},\text{mcs}} \phi;\ \mathcal{S} \mid\!\sim^{\cap}_{\mathfrak{L},\text{mcs}} \psi \ \Rightarrow$
  $\bigcap \text{MCS}_{\mathfrak{L}}(\mathcal{S} \cup \{\psi\}) \vdash \phi;\ \text{MCS}_{\mathfrak{L}}(\mathcal{S} \cup \{\psi\}) = \{\mathcal{T} \cup \{\psi\} \mid \mathcal{T} \in \text{MCS}_{\mathfrak{L}}(\mathcal{S})\} \Rightarrow$
  $\bigcap \text{MCS}_{\mathfrak{L}}(\mathcal{S}), \psi \vdash \phi;\ \bigcap \text{MCS}_{\mathfrak{L}}(\mathcal{S}) \vdash \psi \ \Rightarrow$
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  $\bigcap \text{MCS}_{\mathfrak{L}}(\mathcal{S}) \vdash \phi \ \Rightarrow\ \mathcal{S} \mid\!\sim^{\cap}_{\mathfrak{L},\text{mcs}} \phi$.

- $\mid\!\sim^{\cup}_{\mathfrak{L},\text{mcs}}$ does not satisfy cautious cut:

  Let $\mathcal{S} = \{p \wedge q, \neg p \wedge r\}$. Then: $\text{MCS}_{\mathfrak{L}}(\mathcal{S}) = \{\{p \wedge q\}, \{\neg p \wedge r\}\}$ and $\text{MCS}_{\mathfrak{L}}(\mathcal{S} \cup \{q\}) = \{\{p \wedge q, q\}, \{\neg p \wedge r, q\}\}$.

  Thus: $\mathcal{S} \mid\!\sim^{\cup}_{\mathfrak{L},\text{mcs}} q$ and $\mathcal{S}, q \mid\!\sim^{\cup}_{\mathfrak{L},\text{mcs}} q \wedge r$, but $\mathcal{S} \not\mid\!\sim^{\cup}_{\mathfrak{L},\text{mcs}} q \wedge r$.

2. Consistency Handling

- Two sets $\mathcal{S}_1$ and $\mathcal{S}_2$ are *syntactically disjoint* ($\mathcal{S}_1 \parallel \mathcal{S}_2$), if $\text{Atoms}(\mathcal{S}_1) \cap \text{Atoms}(\mathcal{S}_2) = \emptyset$.
- A set $\mathcal{S}$ s.t. $\text{Atoms}(\mathcal{S}) \subsetneq \text{Atoms}(\mathcal{L})$ is *contaminating* (w.r.t. $\mid\!\sim$), if for every ψ and $\mathcal{S}'$ s.t. $\mathcal{S} \parallel \mathcal{S}'$ we have: $\mathcal{S} \mid\!\sim \psi$ iff $\mathcal{S}, \mathcal{S}' \mid\!\sim \psi$.

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Given a logic $\mathfrak{L} = \langle \mathcal{L}, \vdash \rangle$. Properties of $\mid\!\sim \subseteq 2^{\text{WFF}(\mathcal{L})} \times \text{WFF}(\mathcal{L})$:

- *conservative $\vdash$-consistency:* For every ψ and $\vdash$-consistent set $\mathcal{S}$: $\mathcal{S} \mid\!\sim \psi$ iff $\mathcal{S} \vdash \psi$.
- *pre-paraconsistency:* For every $p \neq q \in \text{Atoms}(\mathcal{L})$: $p, \neg p \not\mid\!\sim q$.
- *non-interference:* If $\mathcal{S}_1 \cup \{\psi\} \parallel \mathcal{S}_2$, then $\mathcal{S}_1 \mid\!\sim \psi$ iff $\mathcal{S}_1, \mathcal{S}_2 \mid\!\sim \psi$.[1]
- *crash-resistance:* there is no $\mid\!\sim$-contaminating set of $\mathcal{L}$-formulas.[1]

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[1] M.Caminada, W.Carnielli, P.Dunne. *Semi-stable semantics*. Journal of Logic and Computation 22(5), pp. 1207-1254, 2011.

Inconsistency Gandling – Summary of the Results

	$\mid\!\sim^{\cap}_{\mathfrak{L},\mathsf{AT},\mathsf{sem}}$	$\mid\!\sim^{\Cap}_{\mathfrak{L},\mathsf{AT},\mathsf{sem}}$	$\mid\!\sim^{\cup}_{\mathfrak{L},\mathsf{AT},\mathsf{sem}}$
conservative $\vdash$-consistency	✓	✓	✓
pre-paraconsistency	✓	✓	✓
non-interference, $\mathsf{AT} \in \{\mathsf{con}, \mathsf{set}\}$	✓	✓	prf, stb, sstb
non-interference, $\mathsf{AT} = \mathsf{dir}$	prf, stb, sstb	prf, stb, sstb	prf, stb, sstb
crash-resistance, $\mathsf{AT} \in \{\mathsf{con}, \mathsf{set}\}$	✓	✓	prf, stb, sstb
crash-resistance, $\mathsf{AT} = \mathsf{dir}$	prf, stb, sstb	prf, stb, sstb	prf, stb, sstb

(✓ = grd, prf, stb, sstb)

For pre-paraconsistency, non-interference and crash-resistance, $\mathfrak{L}$ is also assumed to be uniform: For every two sets of formulas $\mathcal{S}_1, \mathcal{S}_2$ and a formula ϕ such that $\mathcal{S}_2$ is both $\vdash$-consistent and syntactically disjoint from $\mathcal{S}_1 \cup \{\phi\}$, it holds that $\mathcal{S}_1 \vdash \phi$ iff $\mathcal{S}_1, \mathcal{S}_2 \vdash \phi$.

O. Arieli, A. Borg, C. Straßer, *Characterizations and classifications of argumentative entailments*, KR'21, 2021.

Some Further Details: Pre-Paraconsistency

Let $\mathfrak{L}$ be a uniform logic. For every $\mathsf{sem} \in \{\mathsf{grd}, \mathsf{prf}, \mathsf{stb}, \mathsf{sstb}\}$, $\mathsf{AT} \in \{\mathsf{dir}, \mathsf{con}, \mathsf{set}\}$, $\star \in \{\cap, \Cap, \cup\}$: $\mathrel{|\!\sim}^{\star}_{\mathfrak{L},\mathsf{AT},\mathsf{sem}}$ is pre-paraconsistent.

Some Further Details: Pre-Paraconsistency

Let $\mathfrak{L}$ be a uniform logic. For every sem $\in \{\text{grd}, \text{prf}, \text{stb}, \text{sstb}\}$, AT $\in \{\text{dir}, \text{con}, \text{set}\}$, $\star \in \{\cap, \Cap, \cup\}$: $\mid\!\sim^{\star}_{\mathfrak{L},\text{AT},\text{sem}}$ is pre-paraconsistent.

Proof: Let $\Gamma \Rightarrow q \in \text{Arg}_{\mathfrak{L}}(\{p, \neg p\})$. Then $\Gamma \vdash q$. Since $q \notin \text{Atoms}(\Gamma)$, by the uniformity of $\mathfrak{L}$ either $\vdash q$ or Γ is $\vdash$-inconsistent.
The former is excluded by the structurality and non-triviality of $\mathfrak{L}$. Now:

- If AT $\in \{\text{con}, \text{set}\}$: $\Gamma \vdash q$ is attacked by $\Rightarrow \neg \bigwedge \Gamma'$ for $\Gamma' \subseteq \Gamma$, and since the later has no attackers, $\Gamma \Rightarrow q$ cannot be in any extension of the framework. Thus, there is no extension with argument whose conclusion is q, and so $p, \neg p \not\mid\!\sim^{\star}_{\mathfrak{L},\text{AT},\text{sem}} q$.
- If AT $\in \{\text{con}, \text{dir}\}$: By support consistency (for every extension $\mathcal{E}$, $\text{Supps}(\mathcal{E})$ is $\vdash$-consistent), $\Gamma \Rightarrow q$ cannot be in any extension of the framework. Thus, again, there is no extension with argument whose conclusion is q, and so $p, \neg p \not\mid\!\sim^{\star}_{\mathfrak{L},\text{AT},\text{sem}} q$. $\square$

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