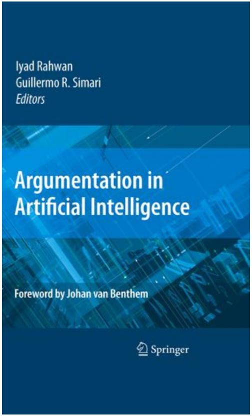
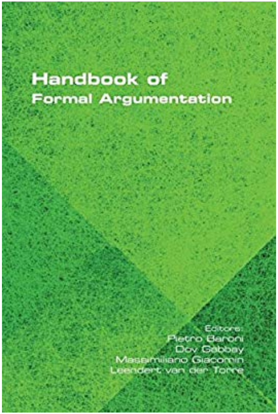
Module 2

Logical Argumentation

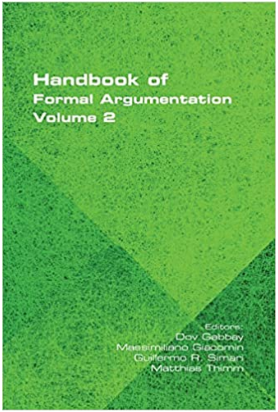

Handbooks on Argumentation Theory



(a) Rahwan, Simari
Springer 2008



(b) Baroni, Gabbay, Giacomin, van der Torre
College Publications, 2018



(c) Gabbay, Giacomin, Simari, Thimm
College Publications, 2021

(There are others. These are the most relevant to this presentation)

Plan of Module 2

1. **Motivation and Introduction**

2. Abstract Argumentation Frameworks
   - Basic Definitions, Semantics
   - The Induced Entailments
   - Paraconsistent Semantics

3. Logical (Deductive) Argumentation Frameworks

4. Some Instantiations
   - Sequent-based Argumentation
   - ASPIC Systems
   - Assumption-based Argumentation

Motivation and Introduction

All the paraconsistent formalisms considered in Module 1 are defined by *logics*. In particular, they are *monotonic*.

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Deductive reasoning should be combined with *defeasible reasoning*.

> *"Human reasoners do not operate with arguments and inferences which allow for absolutely no counterexamples: instead, they typically operate with arguments and inferences whose conclusion is true, or at least highly plausible, in cases where the premises are true and nothing abnormal is going on. The requirement that the conclusion be true in absolutely all situations where the premises are true (including highly unlikely situations) is, for most practical purposes, overkill."*
>
> *(Catharina Dutilh Novaes, The 'built-in opponent'-conception of logic and deduction, 2012)*

Argumentation Theory

Argumentation theory is the interdisciplinary study of how conclusions can be reached through logical reasoning [...]. It includes the arts and sciences of civil debate, dialogue, conversation, and persuasion. It studies rules of inference, logic, and procedural rules in both artificial and real world settings. (Wikipedia)

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Scope of this course:

- Argumentation has been studied since Antiquity.
- We shall hardly discuss here historical or philosophical issues, but:
- Describe formal and computational argumentative methods, used in particular in CS and AI (for paraconsistent, non-monotonic reasoning).

Why Argumentation is Useful for Us?

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  An argumentative perspective on the logical foundations of defeasible reasoning.

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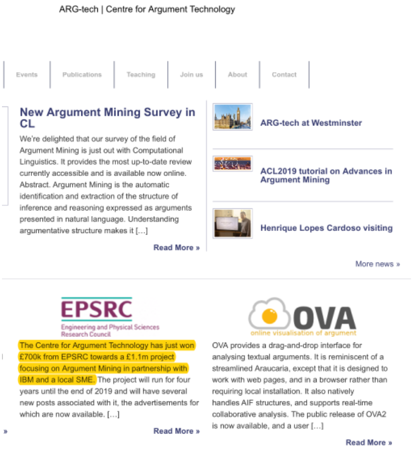
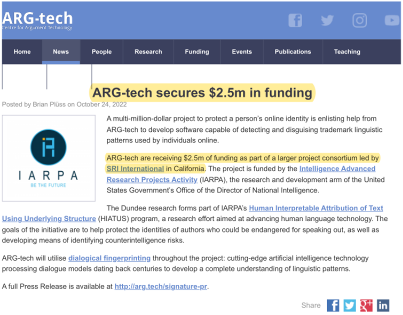
- AI-related applications (argument-mining, debates & discussions, analysis of clinical evidence in medical systems, agent-based negotiations over the web, critical thinking support, etc.)

Handbook of Formal Argumentation, Chapter 14: *Foundations of implementations for formal argumentation* (Cerutti, Gaggl, Thimm, Wallner).

ARG-Tech, Center of Argument technology, University of Dundee (www.arg-tech.org)



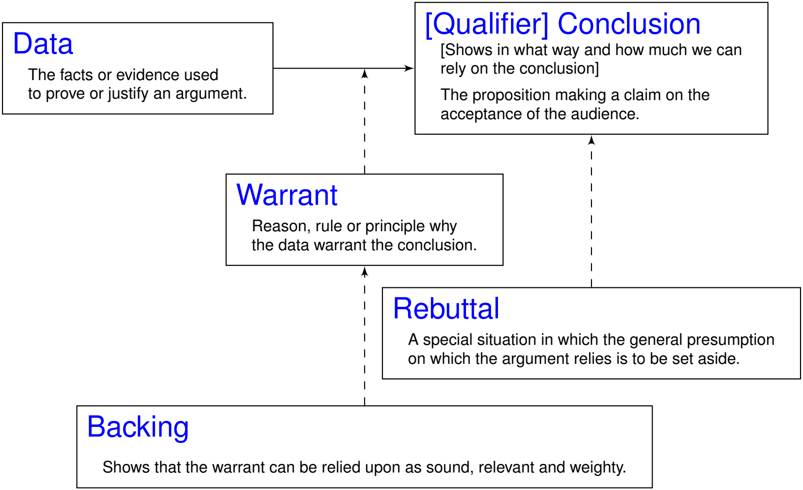


Landmarks in Modern Argumentation Theory

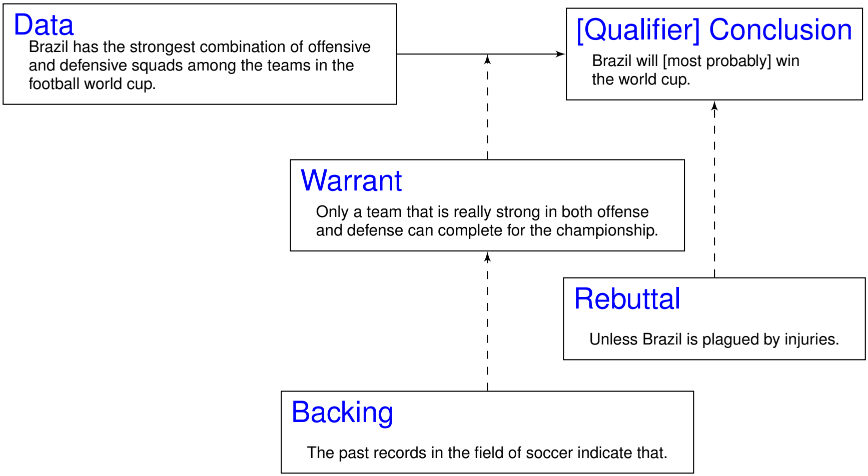
- Stephen E. Toulmin, *The uses of argument*. Cambridge university press, 1958.
- John L. Pollock, *Defeasible reasoning*. Cognitive Science 11, pp. 481–518, 1987.
- Phan Minh Dung, *On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games*. Artificial Intelligence 77(2), pp. 321–357, 1995.

Handbook of Formal Argumentation (volume 1), Chapters 1 & 2:
- *Argumentation theory in formal and computational perspective* (Van Eemeren and Verheij),
- *Historical overview of formal argumentation* (Prakken)

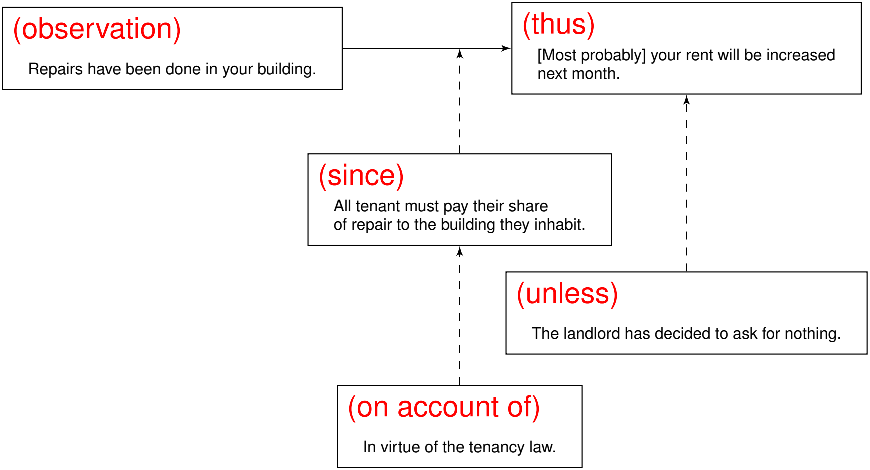
Toulmin Model of Reasoning



Toulmin Model of Reasoning – Example



Toulmin Model of Reasoning – Another Example



When an Arguments is Accepted?
Dung's Abstract Approach

ELSEVIER
Artificial Intelligence 77 (1995) 321–357
Artificial Intelligence

On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and *n*-person games*

Phan Minh Dung*
Division of Computer Science, Asian Institute of Technology, GPO Box 2754, Bangkok 10501, Thailand

Received June 1993; revised April 1994

- An abstract perspective: An argument is an *abstract entity* whose role is solely determined by its relations to other arguments. No special attention is paid to the internal structure of the arguments.
- Whether or not a rational agent believes in a statement depends on whether or not the argument supporting this statement can be successfully defended against the counterarguments.

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Abstract Argumentation Frameworks

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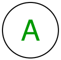

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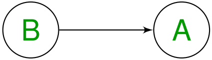

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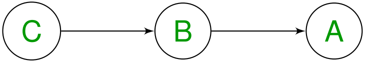

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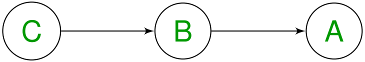
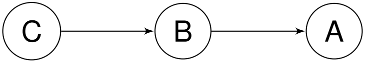


The arguments that A attacks: $A^+ = \{B \mid (A, B) \in \text{Attack}\}$
The arguments that attack A: $A^- = \{B \mid (B, A) \in \text{Attack}\}$
Extensions to sets: $\mathcal{S}^+ = \bigcup_{A \in \mathcal{S}} A^+, \ \mathcal{S}^- = \bigcup_{A \in \mathcal{S}} A^-$
$\mathcal{S}$ attacks A if $A \in \mathcal{S}^+$.

Which Arguments Should be Accepted?



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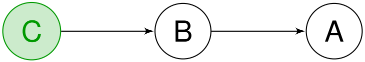

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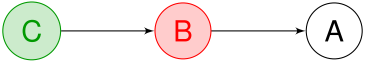

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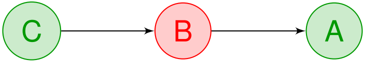

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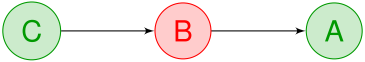

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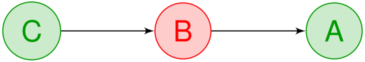

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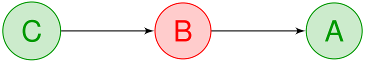

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- $\mathcal{S}$ 'can stand on its own' – any attack on an argument in $\mathcal{S}$ is counter-attacked by $\mathcal{S}$ (that is, $\mathcal{S}$ *defends* all of its elements).
  $\mathcal{S}$ is *admissible*: $\mathcal{S}^- \subseteq \mathcal{S}^+$ (& it is conflict free).

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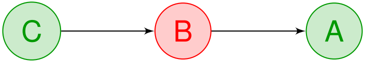

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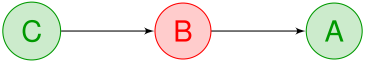

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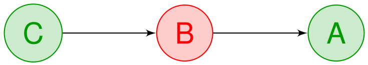

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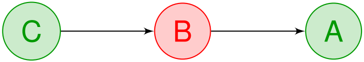
A single complete set: $\{C, A\}$.

Complete Extensions – Further Examples

1. 

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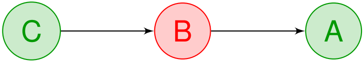
1. 

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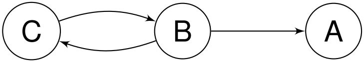
2. 

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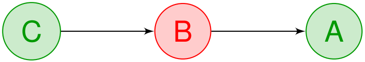
1. 

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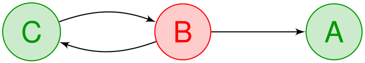
2. 

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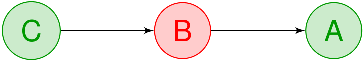
1. 

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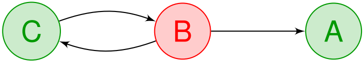
2. 

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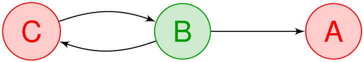

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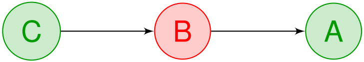
1. 

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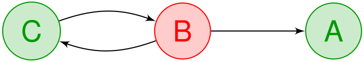
2. 

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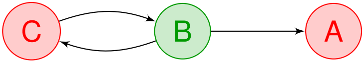

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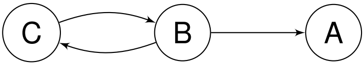

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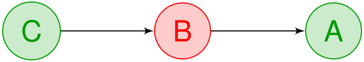
1. 

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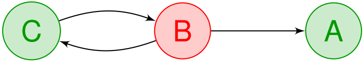
2. 

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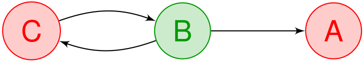

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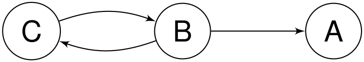

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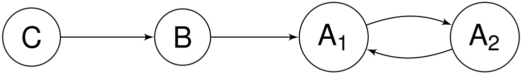
3. 

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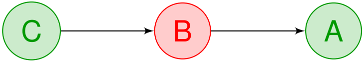
1. 

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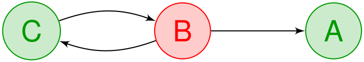
2. 

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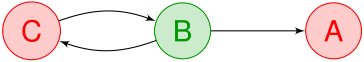

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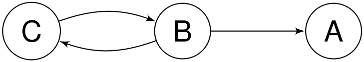

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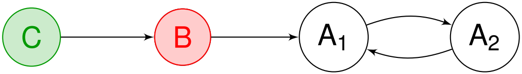
3. 

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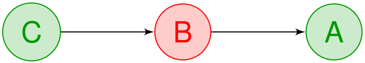
1. 

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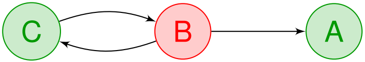
2. 

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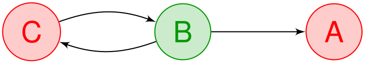

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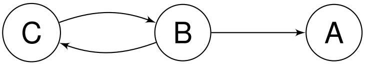
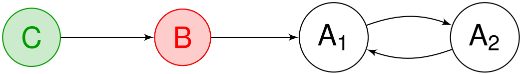
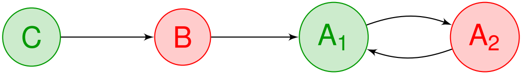


$\{\}$

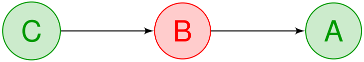
3. 

$\{C\}$

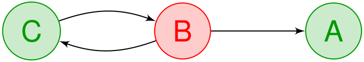


$\{C, A_1\}$

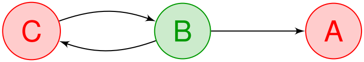
Complete Extensions – Further Examples

1. 

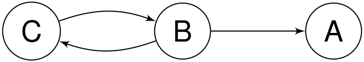
$\{C,A\}$

2. 

$\{C,A\}$



$\{B\}$



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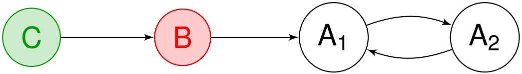
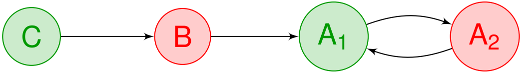
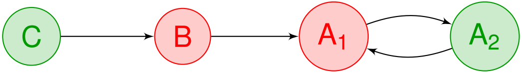
3. 

$\{C\}$



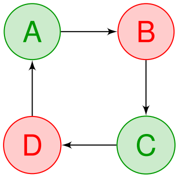
$\{C,A_1\}$

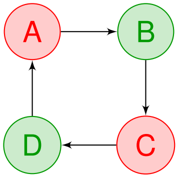


$\{C,A_2\}$

Complete Extensions – Further Examples

- Even-length cycles:





Complete Extensions – Further Examples

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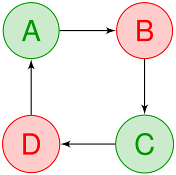
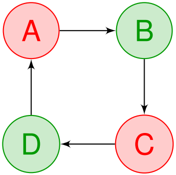




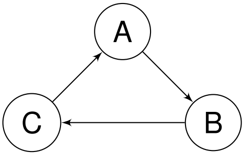
  - The statuses of the arguments are alternating
  - Two complete extensions: $\{A, C\}$ and $\{B, D\}$

Complete Extensions – Further Examples

- Even-length cycles:

  - The statuses of the arguments are alternating
  - Two complete extensions: $\{A, C\}$ and $\{B, D\}$

- Odd-length cycles:



Complete Extensions – Further Examples

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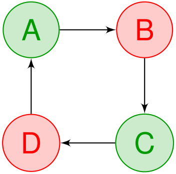

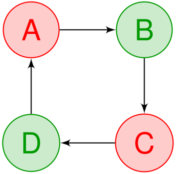

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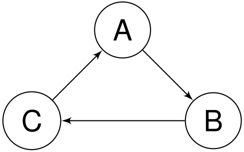

  - No argument belongs to a complete extension:

Complete Extensions – Further Examples

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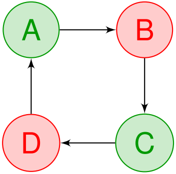

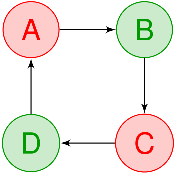

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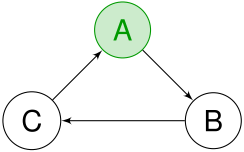
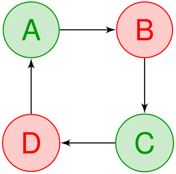
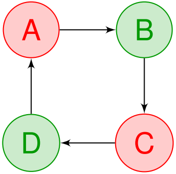


  - No argument belongs to a complete extension:
    If A is accepted,

Complete Extensions – Further Examples

- Even-length cycles:





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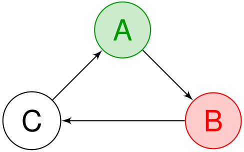
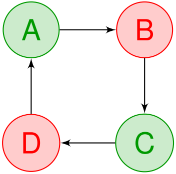
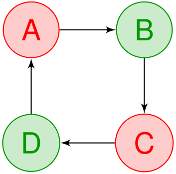


  - No argument belongs to a complete extension:
    If A is accepted, then B is rejected.

Complete Extensions – Further Examples

- Even-length cycles:





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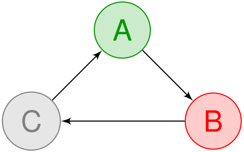

  - No argument belongs to a complete extension:
    If A is accepted, then *B* is rejected. Thus, if C is accepted then {A,C} is not cf., and if C is rejected then {A} is not admissible

Complete Extensions – Further Examples

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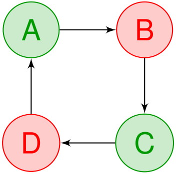

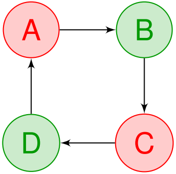

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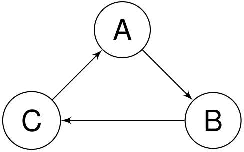
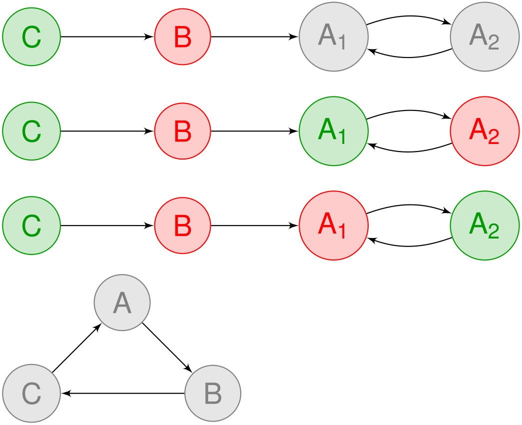
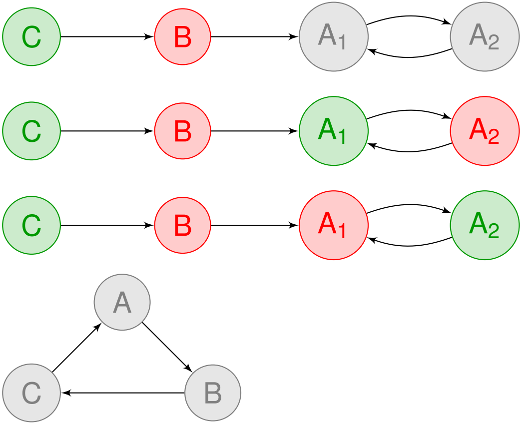


  - No argument belongs to a complete extension:
    If A is accepted, then *B* is rejected. Thus, if C is accepted then {A,C} is not cf., and if C is rejected then {A} is not admissible
  - The only complete extension is the emptyset

Three-Valued Semantics (Labeling)

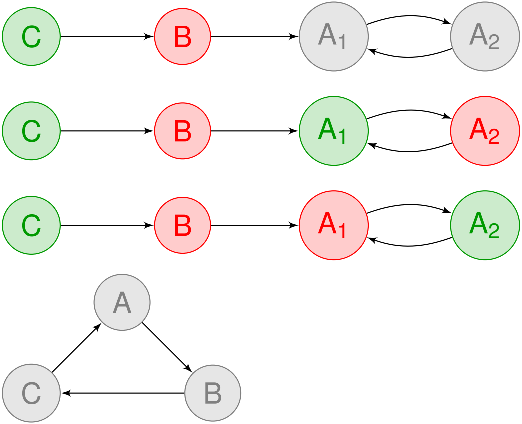


Three-Valued Semantics (Labeling)



- The gray nodes are not accepted, but there is no reason to reject them either: they are not attacked by an accepted argument.
- 3-valued complete labeling: In (accepted), Out (rejected), None.

Three-Valued Semantics (Labeling)



1. An argument is accepted iff <u>all</u> of its attackers are rejected,
2. An argument is rejected iff it has an accepted attacker,
3. Otherwise, the status of the argument is undecided.

Extension-based Vs. Labeling Semantics

- *Complete extension* $\mathcal{E} \subseteq \text{Args}$ of $\mathcal{AF} = \langle \text{Args}, \text{Attack} \rangle$:
  - Conflict free: $\neg \exists A, B \in \mathcal{E}$ such that $(A, B) \in \text{Attack}$,
  - Defends all of its arguments (admissibility): $\mathcal{E} \subseteq \text{Def}(\mathcal{E}) = \{A \in \text{Args} \mid A^- \subseteq \mathcal{E}^+\}$, and
  - Defends only its arguments: $\text{Def}(\mathcal{E}) \subseteq \mathcal{E}$.

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- *Complete labeling* $\text{L} : \text{Args} \to \{\text{in}, \text{out}, \text{none}\}$ of $\mathcal{AF}$:
  - (in): $\text{L}(A) = \text{in} \Rightarrow \forall B \in A^- \; \text{L}(B) = \text{out}$.
  - (out): $\text{L}(A) = \text{out} \Rightarrow \exists B \in A^-$ such that $\text{L}(B) = \text{in}$.
  - (none): $\text{L}(A) = \text{none} \Rightarrow \exists B \in A^- \; \text{L}(B) \neq \text{out} \wedge \forall B \in A^- \; \text{L}(B) \neq \text{in}$.

  (for every argument $A \in \text{Args}$)

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Extensions and labelings are dual semantics (Caminada & Gabbay, Stud Log. v93, 2009)

Complete extensions and complete labeling are one-to-one related:

Extension-based Vs. Labeling Semantics

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Complete extensions and complete labeling are one-to-one related:

- If $\mathcal{E}$ is a complete extension, then $\text{In}(\text{L}) = \mathcal{E}$, $\text{Out}(\text{L}) = \mathcal{E}^+$ and $\text{None}(\text{Args}) = \text{Args} \setminus (\mathcal{E} \cup \mathcal{E}^+)$, is a complete labeling.

Extension-based Vs. Labeling Semantics

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  - (out): $\text{L}(A) = \text{out} \Rightarrow \exists B \in A^-$ such that $\text{L}(B) = \text{in}$.
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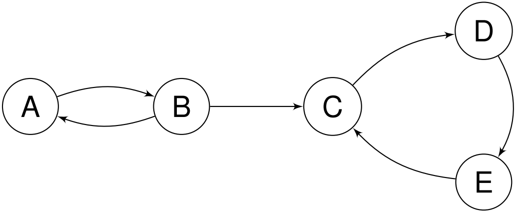
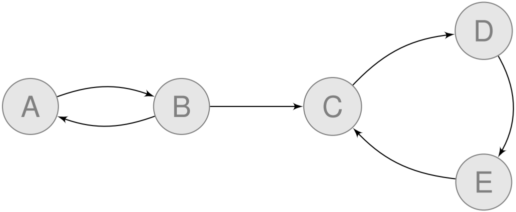
- If L is a complete labeling then $\mathcal{E} = \text{In}(\text{L})$ is a complete extension.

Extension-based Vs. Labeling Semantics

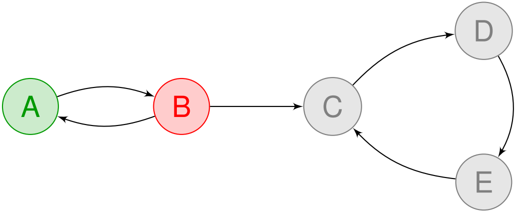


Extension-based Vs. Labeling Semantics



$\mathcal{E}_1 = \emptyset$ $\mathsf{L}_1 = \{A\text{:none}, B\text{:none}, C\text{:none}, D\text{:none}, E\text{:none}\}$

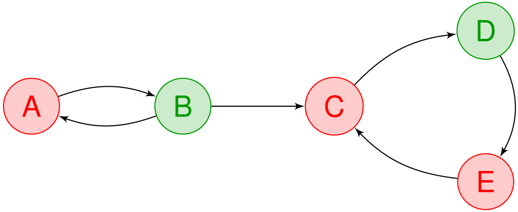
Extension-based Vs. Labeling Semantics



$\mathcal{E}_1 = \emptyset$ $\quad$ $\mathsf{L}_1 = \{A\text{: none}, B\text{: none}, C\text{: none}, D\text{: none}, E\text{: none}\}$

$\mathcal{E}_2 = \{A\}$ $\quad$ $\mathsf{L}_2 = \{A\text{: in}, B\text{: out}, C\text{: none}, D\text{: none}, E\text{: none}\}$

Extension-based Vs. Labeling Semantics



$\mathcal{E}_1 = \emptyset$ $\quad$ $\mathsf{L}_1 = \{A\!:\!\text{none}, B\!:\!\text{none}, C\!:\!\text{none}, D\!:\!\text{none}, E\!:\!\text{none}\}$

$\mathcal{E}_2 = \{A\}$ $\quad$ $\mathsf{L}_2 = \{A\!:\!\text{in}, B\!:\!\text{out}, C\!:\!\text{none}, D\!:\!\text{none}, E\!:\!\text{none}\}$

$\mathcal{E}_3 = \{B, D\}$ $\quad$ $\mathsf{L}_3 = \{A\!:\!\text{out}, B\!:\!\text{in}, C\!:\!\text{out}, D\!:\!\text{in}, E\!:\!\text{out}\}$

Types of Extensions (Dung's Semantics)

A *conflict-free* set $\mathcal{E} \subseteq$ Args is called:

- *naive*, if it is a $\subseteq$-maximal conflict-free subsets of Args,
- *admissible*, if $\mathcal{E} \subseteq$ Def($\mathcal{E}$),
- *complete*, if $\mathcal{E} =$ Def($\mathcal{E}$),
- *grounded*, if it is $\subseteq$-minimal complete extension,
- *preferred*, if it is $\subseteq$-maximal complete extension,
- *stable*, if it is complete & $\mathcal{E} \cup \mathcal{E}^+ =$ Args (attacks anything not in it),
- *semi-stable*, if it is a complete and $\subseteq$-maximal w.r.t. $\mathcal{E} \cup \mathcal{E}^+$ (range).

Other extensions are discussed, e.g., in Chapter 4 of the Handbook of Formal Argumentation: Abstract argumentation frameworks and their semantics (Baroni, Caminada, Giacomin).

Types of Labeling and The Corresponding Extensions

- *Complete extension*: conflict-free extension s.t. $\mathcal{E} = \text{Def}(\mathcal{E})$.
  Complete labeling: 3-val function L satisfying (in), (out), (none).

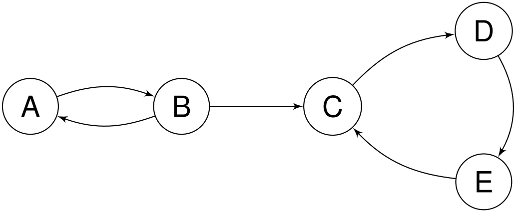
- *Grounded extension*: $\subseteq$-minimal complete extension,
  Complete labeling: complete labeling with $\subseteq$-minimal in-values
  (alternatively, $\subseteq$-minimal out-values, or $\subseteq$-maximal none-values).

- *Preferred extension*: $\subseteq$-maximal complete extension,
  Preferred labeling: complete labeling with $\subseteq$-maximal in-valued
  (alternatively, $\subseteq$-maximal out-values).

- *Stable extension*: complete extension s.t. $\mathcal{E} \cup \mathcal{E}^+ = \text{Args}$,
  Stable labeling: complete labeling without none-values.

- *Semi-stable extension*: complete extension; $\subseteq$-maximal $\mathcal{E} \cup \mathcal{E}^+$.
  Stable labeling: complete labeling with $\subseteq$-minimal none-values.

Example, Revisited



- Grounded extension: $\emptyset$.
  Grounded labeling: $\{A\!:\!\text{none}, B\!:\!\text{none}, C\!:\!\text{none}, D\!:\!\text{none}, E\!:\!\text{none}\}$.

- Preferred extensions: $\{A\}$, $\{B, D\}$.
  Preferred labeling: $\{A\!:\!\text{in}, B\!:\!\text{out}, C\!:\!\text{none}, D\!:\!\text{none}, E\!:\!\text{none}\}$,
  $\{A\!:\!\text{out}, B\!:\!\text{in}, C\!:\!\text{out}, D\!:\!\text{in}, E\!:\!\text{out}\}$.

- (Semi) stable extension: $\{B, D\}$.
  (Semi) stable labeling: $\{A\!:\!\text{out}, B\!:\!\text{in}, C\!:\!\text{out}, D\!:\!\text{in}, E\!:\!\text{out}\}$.

Extensions and Labelings are Dual Semantics

- Extensions $\Rightarrow$ Labelings:

  If $\mathcal{E}$ is a complete (respectively, grounded, preferred, stable, semi-stable) extension,
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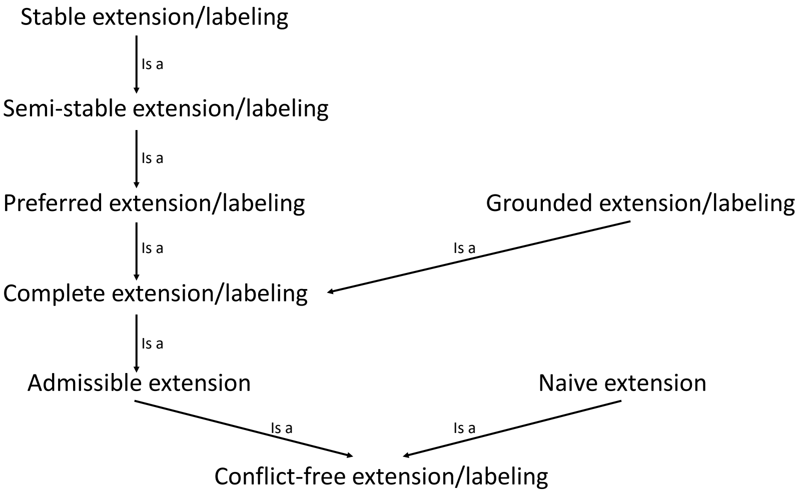
- Labelings $\Rightarrow$ Extensions:

  If L is a complete (respectively, grounded, preferred, stable, semi-stable) labeling,
  then $\mathcal{E} = \text{In}(L)$ is a complete (respectively, grounded, preferred, stable, semi-stable) extension.

M.Caminada, D.Gabbay, A Logical Account of Formal Argumentation. Studia Logica 93(1–2), pp.109–145, 2009.

Relations and Facts



- The grounded extension/labeling is unique.
- Stable extension/labeling do not always exist.

Plan of Module 2

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   - **The Induced Entailments**
   - Paraconsistent Semantics
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4. Some Instantiations
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   - ASPIC Systems
   - Assumption-based Argumentation

Entailments Induced by AAFs

- $\mathcal{AF} = \langle \text{Args}, \text{Attack} \rangle$ – An abstract argumentation framework
- $\text{Sem}(\mathcal{AF})$ – The Sem-extensions of $\mathcal{AF}$
  ($\text{Sem} \in \{\text{Cmp}, \text{Grd}, \text{Prf}, \text{Stb}, \text{SStb}\}$)

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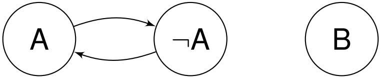

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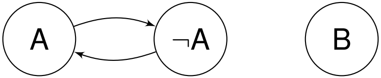

- $\text{Grd} = \{\{B\}\}$, $\text{Prf} = \text{Stb} = \text{SStb} = \{\{A, B\}, \{\neg A, B\}\}$.
- $\forall \text{sem} \in \{\text{cmp}, \text{grd}, \text{prf}, \text{stb}, \text{sstb}\}, \forall \star \in \{\cap, \cup\}$: $\mathcal{AF} \mid\!\sim_{\star\text{sem}} B$.
- $\forall \text{sem} \in \{\text{cmp}, \text{grd}, \text{prf}, \text{stb}, \text{sstb}\}$: $\mathcal{AF} \not\mid\!\sim_{\cap\text{sem}} A$ and $\mathcal{AF} \not\mid\!\sim_{\cap\text{sem}} \neg A$.

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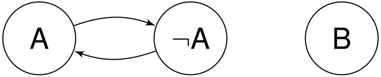

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- Intuitively (& informally), the skeptical entailments are ‘paraconsistent’ in nature.

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Conflict-Tolerant Semantics

A more radical, paraconsistent approach that tolerates conflicts already in the extensions:

- Extensions may not be conflict-free.
- Labelings are four-valued: Args $\rightarrow \{\text{in}, \text{out}, \text{none}, \text{both}\}$.
  ("accepted", "rejected", "undecided", "controversial")

Primary properties:

- A conservative extension of the conflict-free (3-valued) approach: all conflict-free semantics are still obtained and new types of semantics are introduced.
- Any AAF has a nonempty p-complete extension.

O.Arieli. On the acceptance of loops in argumentation frameworks. Journal of Logic and Computation 26(4), 2016.

p-Extensions and p-Labelings

$\mathcal{E} \subseteq$ Args is *paraconsistently admissible* (*p-admissible*), if $\mathcal{E} \subseteq \mathrm{Def}(\mathcal{E})$.

$\mathcal{E} \subseteq$ Args is *paraconsistently complete* (*p-complete*), if $\mathcal{E} = \mathrm{Def}(\mathcal{E})$.

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A 4-val. labeling L is *p-complete*, if it satisfies the following properties:

(pIn) $\text{L}(A) = \text{in} \Leftrightarrow \forall B \in A^- \ \text{L}(B) = \text{out}$

(pOut) $\text{L}(A) = \text{out} \Leftrightarrow \exists B \in A^- \ \text{L}(B) \in \{\text{in}, \text{both}\} \wedge \exists B \in A^- \ \text{L}(B) \in \{\text{in}, \text{none}\}$

(pBoth) $\text{L}(A) = \text{both} \Leftrightarrow \forall B \in A^- \ \text{L}(B) \in \{\text{out}, \text{both}\} \wedge \exists B \in A^- \ \text{L}(B) = \text{both}$

(pNone) $\text{L}(A) = \text{none} \Leftrightarrow \forall B \in A^- \ \text{L}(B) \in \{\text{out}, \text{none}\} \wedge \exists B \in A^- \ \text{L}(B) = \text{none}$

(for every argument $A \in$ Args)

p-Extensions and p-Labelings are Dual Semantics

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From extensions to 4-valued labelings:

$$
ExtLab(\mathcal{E})(A) = \begin{cases} \text{in} & \text{if } A \in \mathcal{E} \text{ and } A \notin \mathcal{E}^+ \\ \text{out} & \text{if } A \notin \mathcal{E} \text{ and } A \in \mathcal{E}^+ \\ \text{both} & \text{if } A \in \mathcal{E} \text{ and } A \in \mathcal{E}^+ \\ \text{none} & \text{if } A \notin \mathcal{E} \text{ and } A \notin \mathcal{E}^+ \end{cases}
$$

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From 4-valued labelings to extensions:

$LabExt(\mathrm{L}) = \mathrm{In}(\mathrm{L}) \bigcup \mathrm{Both}(\mathrm{L}) = \{A \mid L(A) \in \{\text{in}, \text{both}\}\}$ (cf. $\mathcal{D} = \{t, \top\}$ in FDE)

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p-complete extensions and p-complete labelings are 1-to-1 related:

- If $\mathcal{E}$ is a p-complete extension of $\mathcal{AF}$ then $\mathit{ExtLab}(\mathcal{E})$ is a p-complete labeling of $\mathcal{AF}$.
- If L is a p-complete labeling of $\mathcal{AF}$ then $\mathit{LabExt}(\mathrm{L})$ is a p-complete extension for $\mathcal{AF}$.
- $\mathit{ExtLab}(\mathit{LabExt}(\mathrm{L})) = \mathrm{L}$, $\mathit{LabExt}(\mathit{ExtLab}(\mathcal{E})) = \mathcal{E}$.

Conflict-Free and Conflict-Tolerant Semantics

A labeling is called both-free if it has no both assignments (i.e., $\text{Both(L)} = \{A \mid L(A) = \text{both}\} = \emptyset$).

(Conflict-free) complete and (both-free) p-complete extensions & labelings:

- If L is a both-free p-complete labeling for $\mathcal{AF}$, then *LabExt*(L) is a complete extension of $\mathcal{AF}$.
- If $\mathcal{E}$ is a complete extension of $\mathcal{AF}$ then *ExtLab*($\mathcal{E}$) is a both-free p-complete labeling for $\mathcal{AF}$.
- L is a complete labeling for $\mathcal{AF}$ iff it is a both-free p-complete labeling for $\mathcal{AF}$.
- $\mathcal{E}$ is a complete extension of $\mathcal{AF}$ iff it is a conflict-free p-complete extension of $\mathcal{AF}$.

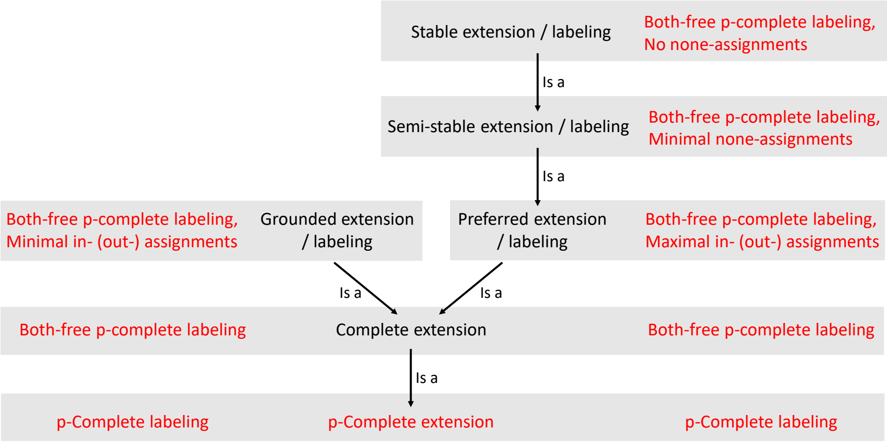
More Relations

A variety of conflict-free semantics for abstract AF may be defined in terms of both-free p-complete labelings. For instance,

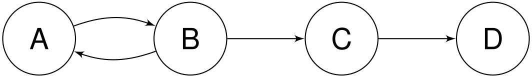
- $\mathcal{E}$ is a grounded extension of $\mathcal{AF}$ iff it is induced by a both-free p-complete labeling L of $\mathcal{AF}$ with $\subseteq$-minimal in-values (alternatively, with $\subseteq$-minimal out-values).
- $\mathcal{E}$ is a preferred extension of $\mathcal{AF}$ iff it is induced by a both-free p-complete labeling L of $\mathcal{AF}$ with $\subseteq$-maximal in-values (alternatively, with $\subseteq$-maximal out-values).
- $\mathcal{E}$ is a semi-stable extension of $\mathcal{AF}$ iff it is induced by a both-free p-complete labeling L of $\mathcal{AF}$ with $\subseteq$-minimal none-values.
- $\mathcal{E}$ is a stable extension of $\mathcal{AF}$ iff it is induced by a both-free p-complete labeling L of $\mathcal{AF}$ without none-values. .

  Thus: $\mathcal{E}$ is a stable extension of $\mathcal{AF}$ iff it is induced by a $\{\text{both}, \text{none}\}$-free p-complete labeling of $\mathcal{AF}$.

Summary of the Semantic Relations



Example



	A	B	C	D	Induced set
1	in	out	in	out	$\{A, C\}$
2	in	out	in	both	$\{A, C, D\}$
3	in	out	none	in	$\{A, D\}$
4	in	out	none	none	$\{A\}$
5	out	in	out	in	$\{B, D\}$
6	out	in	out	none	$\{B\}$
7	out	in	both	out	$\{B, C\}$
8	out	in	both	both	$\{B, C, D\}$

	A	B	C	D	Induced set
9	none	none	in	out	$\{C\}$
10	none	none	in	both	$\{C, D\}$
11	none	none	none	in	$\{D\}$
12	none	none	none	none	$\{\}$
13	both	both	out	in	$\{A, B, D\}$
14	both	both	out	none	$\{A, B\}$
15	both	both	both	out	$\{A, B, C\}$
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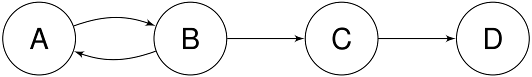
The 4-valued labelings of $\mathcal{AF}$

Example



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The p-complete labelings / extensions of $\mathcal{AF}$

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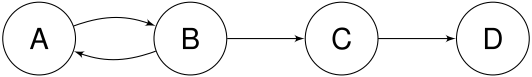

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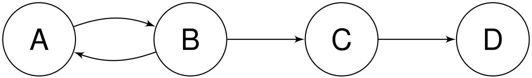

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The stable (and preferred) labelings / extensions of $\mathcal{AF}$

Representations by Theories in Extended FDE

The extensions of $\mathcal{AF} = \langle \text{Args}, \text{Attack} \rangle$ may be represented by theories in Dunn-Belnap logic FDE extended with D'Ottaviano and da Costa's implication ($a \supset b = t$ if $a \in \{f, \perp\}$, otherwise $a \supset b = b$).

The language: an atom for each argument + $\{\neg, \vee, \wedge, \supset, \mathsf{F}\}$.

I. M. D'Ottaviano, and N. C. A. da Costa. Sur un probl'em de Ja ´skowski. C. R.Acad Sc. Paris, Volume 270, S'erie A pp. 1349–1353, 1970.

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Formulas for expressing different sates of arguments: (where not ψ is $\psi \supset \text{F}$):

abbreviation	formula	values
cautiously-accept(a)	a	$t, \top$
cautiously-reject(a)	$\neg a$	$f, \top$
conflicting(a)	cautiously-accept(a) $\wedge$ cautiously-reject(a)	$\top$
coherent(a)	not conflicting(a)	$t, f, \bot$
accept(a)	cautiously-accept(a) $\wedge$ coherent(a)	t
reject(a)	cautiously-reject(a) $\wedge$ coherent(a)	f
undecided(a)	not (cautiously-accept(a) $\vee$ cautiously-reject(a))	$\bot$

E.g., coherent(a) $=$ not conflicting(a) $=$ conflicting(a) $\supset$ F $=$ $(a \wedge \neg a) \supset$ F.

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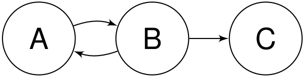
Representations by Theories in Extended FDE

The 4-properties of p-complete labellings are now represented by:

pIn(x)	$\text{accept}(x) \leftrightarrow \bigwedge_{y \in x^-} \text{reject}(y)$
pOut(x)	$\text{reject}(x) \leftrightarrow (\bigvee_{y \in x^-} \text{cautiously-accept}(y) \wedge \bigvee_{y \in x^-} (\text{accept}(y) \vee \text{undecided}(y)))$
pConf(x)	$\text{conflicting}(x) \leftrightarrow (\bigwedge_{y \in x^-} (\text{reject}(y) \vee \text{conflicting}(y)) \wedge \bigvee_{y \in x^-} \text{conflicting}(y))$
pUndec(x)	$\text{undecided}(x) \leftrightarrow (\bigwedge_{y \in x^-} (\text{reject}(y) \vee \text{undecided}(y)) \wedge \bigvee_{y \in x^-} \text{undecided}(y))$

Given $\mathcal{AF} = \langle \text{Args}, \text{Attack} \rangle$, the formula $\psi(a, \mathcal{AF})$ is $\psi(x)$ where x is substituted by the atom a (associated with an argument $A \in$ Args), and where the elements in a^- (and in a^+) are determined by Attack.



pIn($c, \mathcal{AF}$) : $\text{accept}(c) \leftrightarrow \text{reject}(b)$

Representations by Theories in Extended FDE

Representation of the p-complete labellings of $\mathcal{AF} = \langle \text{Args}, \text{Attack} \rangle$:

$$\text{pCMP}(\mathcal{AF}) = \bigcup_{x \in \text{Args}} \text{pIn}(x,\mathcal{AF}) \cup \bigcup_{x \in \text{Args}} \text{pOut}(x,\mathcal{AF}) \cup \bigcup_{x \in \text{Args}} \text{pConf}(x,\mathcal{AF}) \cup \bigcup_{x \in \text{Args}} \text{pUndec}(x,\mathcal{AF})$$

Representations by Theories in Extended FDE

Representation of the p-complete labellings of $\mathcal{AF} = \langle \text{Args}, \text{Attack} \rangle$:

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Proposition

Given $\mathcal{AF} = \langle \text{Args}, \text{Attack} \rangle$, there is a correspondence between the 4-valued models of $\text{pCMP}(\mathcal{AF})$, the 4-states p-complete labelings of $\mathcal{AF}$, and the p-complete extensions of $\mathcal{AF}$:

- *if ν is a model of $\text{pCMP}(\mathcal{AF})$ then $\text{ValLal}(\nu)$ is a p-complete labeling of $\mathcal{AF}$ and $\text{LalExt}(\text{ValLab}(\nu))$ is a p-complete extension of $\mathcal{AF}$.*
- *If L is a p-complete labeling of $\mathcal{AF}$ then $\text{LalVal}(\text{L})$ is a model of $\text{pCMP}(\mathcal{AF})$ and $\text{LalExt}(\text{L})$ is a p-complete extension of $\mathcal{AF}$.*
- *If $\mathcal{E}$ is a p-complete extension of $\mathcal{AF}$ then $\text{ExtLab}(\mathcal{E})$ is a p-complete labeling of $\mathcal{AF}$ and $\text{LalVal}(\text{EextLab}(\mathcal{E}))$ is a model of $\text{pCMP}(\mathcal{AF})$.*

O.Arieli. Conflict-free and conflict-tolerant semantics for constrained argumentation frameworks. Journal of Applied Logic 13(4):582–604, 2015.

Representations of Other Extensions/Labellings

- p-complete labeling:

pCMP($\mathcal{AF}$)

- complete labeling:

CMP($\mathcal{AF}$) $=$ pCMP($\mathcal{AF}$)$\cup\{$accept(x)$\vee$reject(x)$\vee$undecided(x) $\mid x \in$ Args$\}$.

- stable labeling:

STB($\mathcal{AF}$) $=$ pCMP($\mathcal{AF}$) $\cup$ $\{$accept(x) $\vee$ reject(x) $\mid$ x $\in$ Args$\}$.

Representations of Other Extensions/Labellings

- p-complete labeling:

pCMP($\mathcal{AF}$)

- complete labeling:

$\text{CMP}(\mathcal{AF}) = \text{pCMP}(\mathcal{AF}) \cup \{\text{accept}(x) \vee \text{reject}(x) \vee \text{undecided}(x) \mid x \in \text{Args}\}$.

- stable labeling:

$\text{STB}(\mathcal{AF}) = \text{pCMP}(\mathcal{AF}) \cup \{\text{accept}(x) \vee \text{reject}(x) \mid x \in \text{Args}\}$.

For grounded and preferred labeling/extensions, we need to express also minimality/maximality conditions. We do so by incorporating Quantified Boolean Formulas (QBFs), namely: extending the propositional language with universal and existential quantifiers $\forall, \exists$ over propositional variables..

Representations of Other Extensions/Labellings

- p-complete labeling:

$\mathrm{pCMP}(\mathcal{AF})$

- complete labeling:

$\mathrm{CMP}(\mathcal{AF}) = \mathrm{pCMP}(\mathcal{AF}) \cup \{\mathrm{accept}(x) \vee \mathrm{reject}(x) \vee \mathrm{undecided}(x) \mid x \in \mathrm{Args}\}$.

- stable labeling:

$\mathrm{STB}(\mathcal{AF}) = \mathrm{pCMP}(\mathcal{AF}) \cup \{\mathrm{accept}(x) \vee \mathrm{reject}(x) \mid x \in \mathrm{Args}\}$.

For grounded and preferred labeling/extensions, we need to express also minimality/maximality conditions. We do so by incorporating Quantified Boolean Formulas (QBFs), namely: extending the propositional language with universal and existential quantifiers $\forall, \exists$ over propositional variables..

Example: $\exists x \forall y \psi$ means that there exists a truth assignment for x such that for every truth assignment for y, the formula ψ is true.
(where, e.g., $\forall x \psi$ stands for $\psi[\mathrm{T}/x] \wedge \psi[\mathrm{F}/x] \wedge \psi[\mathrm{B}/x] \wedge \psi[\mathrm{N}/x]$).

Representations of Other Extensions/Labellings

$\mathrm{Min}_t(\mathrm{pCMP}(\mathcal{AF}))$: minimization of t assignments in p-complete labelings:

$$\forall x_1,\ldots,x_n \Big(\bigwedge_{a_i \in \mathrm{Args}} \mathrm{pCMP}(\mathcal{AF})\Big[x_1/a_1,\ldots,x_n/a_n\Big] \supset$$

$$\Big(\bigwedge_{a_i \in \mathrm{Args},\ 1 \le i \le n} \Big(\mathrm{accept}(x_i) \supset \mathrm{accept}(a_i)\Big) \supset$$

$$\bigwedge_{a_i \in \mathrm{Args},\ 1 \le i \le n} \Big(\mathrm{accept}(a_i) \supset \mathrm{accept}(x_i)\Big)\Big) \Big)$$

$\mathrm{Max}_t(\mathrm{pCMP}(\mathcal{AF}))$: maximization of t assignments in p-complete labelings:

$$\forall x_1,\ldots,x_n \Big(\bigwedge_{a_i \in \mathrm{Args}} \mathrm{pCMP}(\mathcal{AF})\Big[x_1/a_1,\ldots,x_n/a_n\Big] \supset$$

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Representations of Other Extensions/Labellings

- p-grounded labeling:

$\text{pGRD}(\mathcal{AF}) = \text{pCMP}(\mathcal{AF}) \cup \{\text{Min}_t(\text{pCMP}(\mathcal{AF}))\}$.

- p-perferred labeling:

$\text{pPRF}(\mathcal{AF}) = \text{pCMP}(\mathcal{AF}) \cup \{\text{Max}_t(\text{pCMP}(\mathcal{AF}))\}$.

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Proposition

Given $\mathcal{AF} = \langle \text{Args}, \text{Attack} \rangle$.

- *There is a correspondence between the 4-valued models of* $\text{pGRD}(\mathcal{AF})$*, the p-grounded labelings of* $\mathcal{AF}$*, and the p-grounded extensions of* $\mathcal{AF}$*.*
- *There is a correspondence between the 4-valued models of* $\text{pPRF}(\mathcal{AF})$*, the p-preferred labelings of* $\mathcal{AF}$*, and the p-preferred extensions of* $\mathcal{AF}$*.*

Similar results are obtained for stable, semi-stable extensions/labellings and the corresponding semantics for the 3-valued case.

Further References

OBF-theories for representing semantics of abstract argumentation frameworks:

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A survey on other logical theories for standard, 3-valued semantics of abstract argumentation:

P. Besnard, C. Cayrol, M. Lagasquie-Schiex. Logical theories and abstract argumentation: A survey of existing works, Journal of Argument & Computation 11(1–2): 41–102, 2020.

A survey on computation methods and Implementations:

F. Cerutti, S. A. Gaggl, M. Thimm, J. P. Wallne. Foundations of implementations for formal argumentation, Journal of Applied Logics, IFCoLog Journal of Logics and their Applications 4(8): 2623–2706, 2017.

Plan of Module 2

1. Motivation and Introduction
2. Abstract Argumentation Frameworks (AAFs)
   - Basic Definitions, Semantics
   - The Induced Entailments
   - Paraconsistent Semantics
3. **Logical (Deductive) Argumentation Frameworks**
4. Some Instantiations
   - Sequent-based Argumentation
   - ASPIC Systems
   - Assumption-based Argumentation

Logical (Deductive) Argumentation

[Simari & Loui, 1992, Besnard & Hunter, 2001]

Arguments are not just arbitrary abstract entities, but represent explicit *inferences* (based on some underlying *logic*).

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Definition (Besnard & Hunter)

A BH-argument based on $\mathcal{S}$ is a pair $A = \langle \Gamma, \psi \rangle$, where:

- $\mathcal{S}$ (*the set of assertions; background knowledge*),
- Γ (the *support set*) – finite sets of propositional formulas,
- ψ (the *conclusion*) – a propositional formula,
- Γ is a minimally consistent subset of $\mathcal{S}$ such that $\Gamma \vdash_{\mathsf{CL}} \psi$.

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Notation: $\mathsf{Args}_{\mathsf{BH}}(\mathcal{S})$ – the set of the $\mathcal{S}$-based BH-arguments.

What is a Logical Argument?

Are all the restrictions on logical arguments really needed?

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- Extensions to arbitrary (propositional) languages
- Extensions to arbitrary (effectively computable) logics
- The support sets need not be minimal
- The support sets need not be consistent

What is a Logical Argument?

1. The Underlying Logic Need Not Be Classical Logic

Recall from Module 1:

A (Tarskian) *consequence relation* $\vdash$ for a language $\mathcal{L}$:

- Reflexivity: $\psi \vdash \psi$.
- Monotonicity: if $\mathcal{S} \vdash \psi$ and $\mathcal{S} \subseteq \mathcal{S}'$, then $\mathcal{S}' \vdash \psi$.
- Transitivity: if $\mathcal{S} \vdash \psi$ and $\mathcal{S}', \psi \vdash \varphi$ then $\mathcal{S}, \mathcal{S}' \vdash \varphi$.

A consequence relation $\vdash$ is called:

- Structural: if $\mathcal{S} \vdash \psi$ then $\theta(\Gamma) \vdash \theta(\psi)$ for every $\mathcal{L}$-substitution θ.
- Non-trivial: $\mathcal{S} \nvdash \psi$ for some $\mathcal{S} \neq \emptyset$.
- Finitary: if $\mathcal{S} \vdash \psi$ then $\Gamma \vdash \psi$ for some finite $\Gamma \subseteq \mathcal{S}$.

A (propositional) *logic* is a pair $\mathfrak{L} = \langle \mathcal{L}, \vdash \rangle$, where

- $\mathcal{L}$ is a propositional language, and
- $\vdash$ is a structural, non-trivial and finitary CR for $\mathcal{L}$.

2. The Language Need Not Be The Standard Propositional One

Recall from Module 1:

What is a Logical Argument?

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Recall from Module 1:

$\wedge$ is a *conjunction* for $\mathfrak{L}$ if $\mathcal{S} \vdash \psi \wedge \varphi$ iff $\mathcal{S} \vdash \psi$ and $\mathcal{T} \vdash \varphi$.

$\vee$ is a *disjunction* for $\mathfrak{L}$ if $\mathcal{S}, \psi \vee \varphi \vdash \sigma$ iff $\mathcal{S}, \psi \vdash \sigma$ and $\mathcal{S}, \varphi \vdash \sigma$.

$\supset$ is an *implication* for $\mathfrak{L}$ if $\mathcal{S}, \varphi \vdash \psi$ iff $\mathcal{S} \vdash \varphi \supset \psi$.

$\neg$ is a (weak) *negation* for $\mathfrak{L}$ if $p \nvdash \neg p$ and $\neg p \nvdash p$.

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$\wedge$ is a *conjunction* for $\mathfrak{L}$ if $\mathcal{S} \vdash \psi \wedge \varphi$ iff $\mathcal{S} \vdash \psi$ and $\mathcal{T} \vdash \varphi$.
(Equivalently, $\Gamma, \psi \wedge \phi \vdash \tau \ \Leftrightarrow \Gamma, \psi, \phi \vdash \tau$)

$\vee$ is a *disjunction* for $\mathfrak{L}$ if $\mathcal{S}, \psi \vee \varphi \vdash \sigma$ iff $\mathcal{S}, \psi \vdash \sigma$ and $\mathcal{S}, \varphi \vdash \sigma$.
(Equivalently, if $\vdash$ is multi-conclusioned, $\Gamma \vdash \psi \vee \phi \ \Leftrightarrow \Gamma \vdash \psi, \phi$)

$\supset$ is an *implication* for $\mathfrak{L}$ if $\mathcal{S}, \varphi \vdash \psi$ iff $\mathcal{S} \vdash \varphi \supset \psi$.
(Inferences to theoremhood: $\psi_1, \ldots, \psi_n \vdash \phi \ \Leftrightarrow \vdash \psi_1 \supset (\psi_2 \ldots \supset (\psi_n \supset \phi))$)

$\neg$ is a (weak) *negation* for $\mathfrak{L}$ if $p \nvdash \neg p$ and $\neg p \nvdash p$.
(A stronger condition: $\neg$-containment in / coherence with classical logic)

3. The Support Set Need Not Be Minimal

- Mathematical proofs need not be based on minimal assumptions.
- Minimality may not be desirable (e.g., for majority votes).
- $\{p, q\}$ is a *stronger* support for $p \vee q$ than only $\{p\}$ (or $\{q\}$).

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4. The Support Set Need Not Be Consistent

- Paraconsistent logics properly handle inconsistent information.
- Computational considerations: Deciding the existence of a minimally consistent subset of formulas implying a consequent is at the second level of the polynomial hierarchy.

What is a Logical Argument?

Thus, what really matters for an argument, is that

- its consequent would *logically follow* from the support set, and
- there would be an *effective way* of constructing and identifying it.

Arguments are syntactical objects that are

- *effectively computable* by a formal proof system (logic related)
- *refutable* by the attack relation of the argumentation system.

What is a Logical Argument?

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A proper way of representing logical arguments is by (the proof theoretical notion of) sequents.

Gentzen, G.: Untersuchungen über das logische Schliessen. Mathematische Zeitschrift 39:176–210, 1934

Arguments as Sequents

A *proof-theoretic view* of arguments:

Given a logic $\mathfrak{L} = \langle \mathcal{L}, \vdash \rangle$, *logical arguments* are defined as follows:

- *$\mathcal{L}$-sequent*: expression $\Gamma \Rightarrow \Delta$ (Γ, Δ – finite sets; $\Rightarrow \notin \mathcal{L}$).
- *$\mathfrak{L}$-argument*: $\mathcal{L}$-sequent $\Gamma \Rightarrow \psi$, where $\Gamma \vdash \psi$.
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- *Support*: $\Gamma = \text{Sup}(\Gamma \Rightarrow \psi)$, *Conclusion*: $\psi = \text{Con}(\Gamma \Rightarrow \psi)$.
- $\text{Arg}_{\mathfrak{L}}(\mathcal{S})$: the set of the $\mathcal{S}$-based $\mathfrak{L}$-arguments

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Example

$\mathfrak{L} = \mathrm{CL}$ (classical logic), $\mathcal{S} = \{p, \neg p, q\}$.

$\mathrm{Arg}_{\mathrm{CL}}(\mathcal{S}) = \{p \Rightarrow p \quad p, q \Rightarrow p \wedge q \quad \Rightarrow p \vee \neg p \quad p, \neg p \Rightarrow q \quad \ldots\}$.

Gentzen, G.: Untersuchungen über das logische Schliessen. Mathematische Zeitschrift 39:176–210, 1934

Construction of Arguments

Standard *sequent calculi* are used to construct arguments from simpler arguments, by means of ***inference rules***:

$$\frac{\Gamma_1 \Rightarrow \Delta_1 \quad \dots \quad \Gamma_n \Rightarrow \Delta_n}{\Gamma \Rightarrow \Delta}$$

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$$\frac{\Gamma_1 \Rightarrow \Delta_1 \quad \ldots \quad \Gamma_n \Rightarrow \Delta_n}{\Gamma \Rightarrow \Delta}$$

Example (Rules taken from the sequent calculus LK for CL)

$$\frac{\Gamma \Rightarrow \Delta, \psi}{\Gamma, \neg\psi \Rightarrow \Delta} \ (\neg\Rightarrow) \qquad \frac{\Gamma, \psi \Rightarrow \Delta}{\Gamma \Rightarrow \Delta, \neg\psi} \ (\Rightarrow\neg)$$

$$\frac{\Gamma, \psi, \varphi \Rightarrow \Delta}{\Gamma, \psi \wedge \varphi \Rightarrow \Delta} \ (\wedge\Rightarrow) \qquad \frac{\Gamma \Rightarrow \phi, \Delta \quad \Gamma \Rightarrow \psi, \Delta}{\Gamma \Rightarrow \phi \wedge \psi, \Delta} \ (\Rightarrow\wedge)$$

$$\frac{\Gamma, \psi \Rightarrow \Delta \quad \Gamma, \varphi \Rightarrow \Delta}{\Gamma, \psi \vee \varphi \Rightarrow \Delta} \ (\vee\Rightarrow) \qquad \frac{\Gamma \Rightarrow \Delta, \psi, \varphi}{\Gamma \Rightarrow \Delta, \psi \vee \varphi} \ (\Rightarrow\vee)$$

$$\frac{\Gamma \Rightarrow \psi, \Delta \quad \Gamma, \varphi \Rightarrow \Delta}{\Gamma, \psi \supset \varphi \Rightarrow \Delta} \ (\supset\Rightarrow) \qquad \frac{\Gamma, \psi \Rightarrow \varphi, \Delta}{\Gamma \Rightarrow \psi \supset \varphi, \Delta} \ (\Rightarrow\supset)$$

Derivation Trees

A derivation tree in *LK*

$$
\dfrac{
  \dfrac{\dfrac{\dfrac{\varphi \Rightarrow \varphi}{\varphi \Rightarrow \neg\psi, \varphi}\,[W]}{\Rightarrow \neg\psi, \neg\varphi, \varphi}\,[\Rightarrow\neg]}{\Rightarrow \neg\psi \vee \neg\varphi, \varphi}\,[\Rightarrow\vee]
  \qquad
  \dfrac{\dfrac{\dfrac{\psi \Rightarrow \psi}{\psi \Rightarrow \neg\varphi, \psi}\,[W]}{\Rightarrow \neg\psi, \neg\varphi, \psi}\,[\Rightarrow\neg]}{\Rightarrow \neg\psi \vee \neg\varphi, \psi}\,[\Rightarrow\vee]
}{\Rightarrow \neg\psi \vee \neg\varphi, \psi \wedge \varphi}\,[\Rightarrow\wedge]
$$

$$
\dfrac{\dfrac{\Rightarrow \neg\psi \vee \neg\varphi, \psi \wedge \varphi}{\neg(\psi \wedge \varphi) \Rightarrow \neg\psi \vee \neg\varphi}\,[\neg\Rightarrow]}{\Rightarrow \neg(\psi \wedge \varphi) \supset \neg\psi \vee \neg\varphi}\,[\Rightarrow\supset]
$$

Attacks as Elimination Rules

Attacks (conflicts) between arguments are represented by *sequent elimination (attack) rules*:

$$\frac{\overbrace{\Gamma_1 \Rightarrow \Delta_1}^{\text{attacker}} \quad \overbrace{\ldots}^{\text{conditions}} \quad \overbrace{\Gamma_n \Rightarrow \Delta_n}^{\text{attacked}}}{\underbrace{\Gamma_n \not\Rightarrow \Delta_n}_{\text{eliminated argument}}}$$

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Attack by Undercut

$$\frac{\Gamma_1 \Rightarrow \psi_1 \quad \psi_1 \Rightarrow \neg\bigwedge\Gamma'_2 \quad \neg\bigwedge\Gamma'_2 \Rightarrow \psi_1 \quad \Gamma_2, \Gamma'_2 \Rightarrow \psi_2}{\Gamma_2, \Gamma'_2 \not\Rightarrow \psi_2} \ \text{Ucut}$$

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Attack by Undercut

$$\frac{\Gamma_1 \Rightarrow \psi_1 \quad \psi_1 \Rightarrow \neg\bigwedge\Gamma_2' \quad \neg\bigwedge\Gamma_2' \Rightarrow \psi_1 \quad \Gamma_2, \Gamma_2' \Rightarrow \psi_2}{\Gamma_2, \Gamma_2' \not\Rightarrow \psi_2} \ \text{Ucut}$$

Example

$\mathfrak{L} = \text{CL}, \ \mathcal{S} = \{p, \neg p, q\}$.

The $\mathcal{S}$-argument $\neg p \Rightarrow \neg p$ Ucut-attacks the $\mathcal{S}$-argument $p \Rightarrow p$, as well as $p, q \Rightarrow p \wedge q$.

Some Common Attacks Rules

Undercut

$$\frac{\Gamma_1 \Rightarrow \psi_1 \quad \psi_1 \Rightarrow \neg \bigwedge \Gamma_2' \quad \neg \bigwedge \Gamma_2' \Rightarrow \psi_1 \quad \Gamma_2, \Gamma_2' \Rightarrow \psi_2}{\Gamma_2, \Gamma_2' \not\Rightarrow \psi_2} \text{ Ucut}$$

Direct Undercut

$$\frac{\Gamma_1 \Rightarrow \psi_1 \quad \psi_1 \Rightarrow \neg \gamma_2 \quad \neg \gamma_2 \Rightarrow \psi_1 \quad \Gamma_2, \gamma_2 \Rightarrow \psi_2}{\Gamma_2, \gamma_2 \not\Rightarrow \psi_2} \text{ DirUcut}$$

Defeat

$$\frac{\Gamma_1 \Rightarrow \psi_1 \quad \psi_1 \Rightarrow \neg \bigwedge \Gamma_2' \quad \Gamma_2, \Gamma_2' \Rightarrow \psi_2}{\Gamma_2, \Gamma_2' \not\Rightarrow \psi_2} \text{ Def}$$

Direct Defeat

$$\frac{\Gamma_1 \Rightarrow \psi_1 \quad \psi_1 \Rightarrow \neg \gamma_2 \quad \Gamma_2, \gamma_2 \Rightarrow \psi_2}{\Gamma_2, \gamma_2 \not\Rightarrow \psi_2} \text{ DirDef}$$

Some Common Attacks Rules (Cont'd.)

Rebuttal

$$\frac{\Gamma_1 \Rightarrow \psi_1 \quad \psi_1 \Rightarrow \neg\psi_2 \quad \neg\psi_2 \Rightarrow \psi_1 \quad \Gamma_2 \Rightarrow \psi_2}{\Gamma_2 \not\Rightarrow \psi_2} \text{ Reb}$$

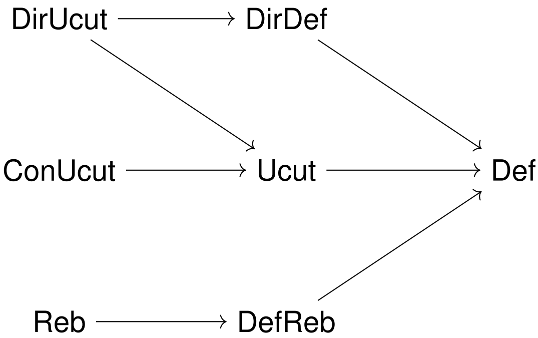
Defeating Rebuttal

$$\frac{\Gamma_1 \Rightarrow \psi_1 \quad \psi_1 \Rightarrow \neg\psi_2 \quad \Gamma_2 \Rightarrow \psi_2}{\Gamma_2 \not\Rightarrow \psi_2} \text{ DefReb}$$

Consistency Undercut

$$\frac{\Rightarrow \neg\bigwedge\Gamma_2 \qquad \Gamma_2, \Gamma_2' \Rightarrow \psi_2}{\Gamma_2, \Gamma_2' \not\Rightarrow \psi_2} \text{ ConUcut}$$

Attacks Rules (Cont'd.)



An arrow from $\mathcal{R}_1$ to $\mathcal{R}_2$ means that $\mathcal{R}_1 \subseteq \mathcal{R}_2$.

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Sequent-based Argumentation Frameworks

A *sequent-based argumentation framework* for $\mathcal{S}$, based on a logic $\mathfrak{L}$ and a set $\mathcal{A}$ of attack rules, is an abstract argumentation framework of the form $\mathcal{AF}(\mathcal{S}) = \langle \mathsf{Arg}_{\mathfrak{L}}(\mathcal{S}), \mathsf{Attack}(\mathcal{A}) \rangle$, where:

- $\mathsf{Arg}_{\mathfrak{L}}(\mathcal{S})$ is the set of the $\mathcal{S}$-based $\mathfrak{L}$-arguments, and
- $(A_1, A_2) \in \mathsf{Attack}(\mathcal{A})$ iff A_1 $\mathcal{R}$-attacks A_2, for some $\mathcal{R} \in \mathcal{A}$.

O.Arieli, C.Straßer, *Sequent-based logical argumentation*, Argument and Computation 6(1):73–99, 2015.

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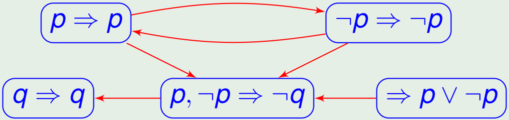
Example

Part of the sequent-based logical argumentation framework for $\mathcal{S} = \{p, \neg p, q\}$, based on classical logic and Undercut:



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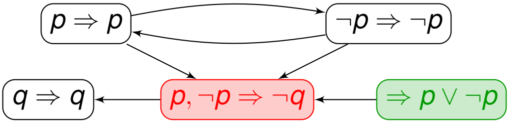
Dung-style Semantics, Revisited

Since sequent-based frameworks are a particular case of abstract argumentation frameworks, Dung's semantics is defined for them.



- $\mathcal{S}$ *attacks* an argument A if there is an $s \in \mathcal{S}$ such that s attacks A

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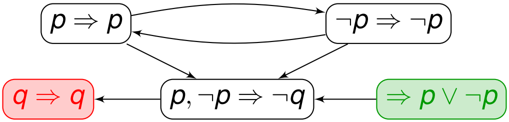

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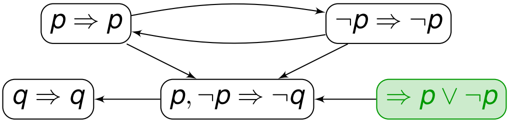

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- $\mathcal{S}$ is *conflict-free* if $\mathcal{S}$ does not attack any of its arguments

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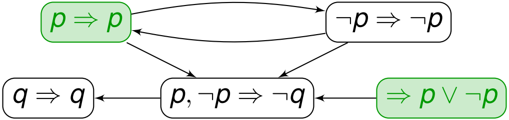

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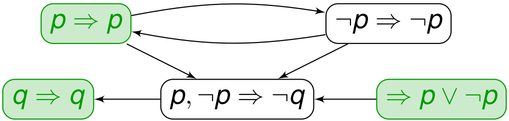
- $\mathcal{S}$ is *conflict-free* if $\mathcal{S}$ does not attack any of its arguments
- $\mathcal{S}$ is an *admissible extension* if it is conflict-free and defends all its arguments

Dung-style Semantics, Revisited

Since sequent-based frameworks are a particular case of abstract argumentation frameworks, Dung's semantics is defined for them.



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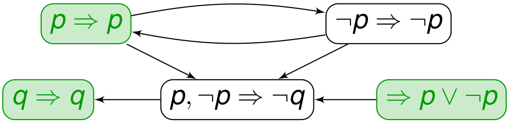

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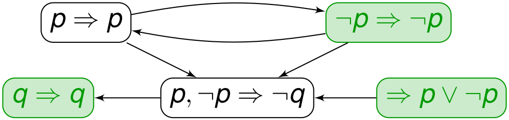

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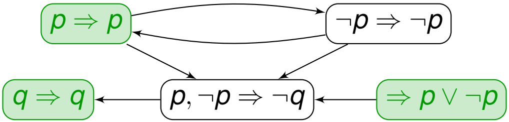

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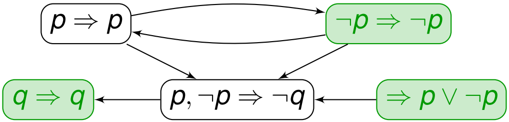

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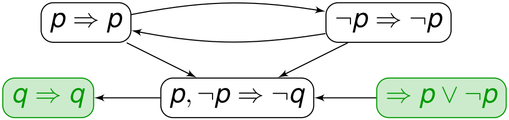

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Another Example

$$\mathfrak{L} = \text{CL},\ \mathcal{R} = \{\text{DirDef}, \text{ConUcut}\},\ \mathcal{S} = \{p, q, \neg p \vee \neg q, r\}$$

$A_1 = r \Rightarrow r$ | $A_4 = \neg p \vee \neg q \Rightarrow \neg p \vee \neg q$ | $A_7 = p, q \Rightarrow p \wedge q$

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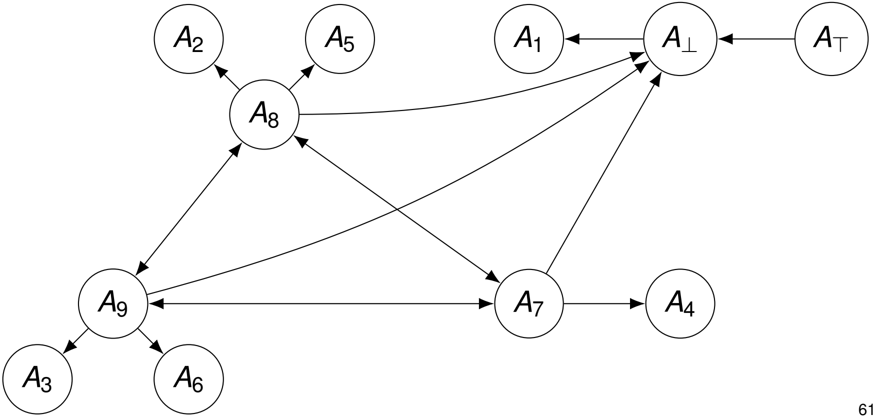

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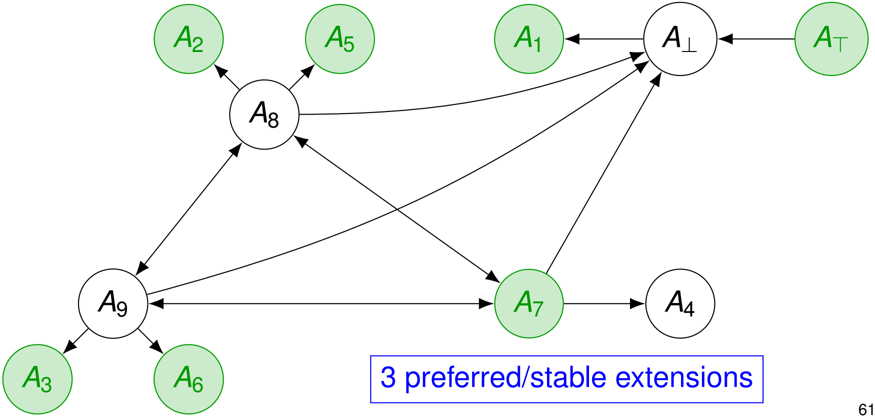

3 preferred/stable extensions

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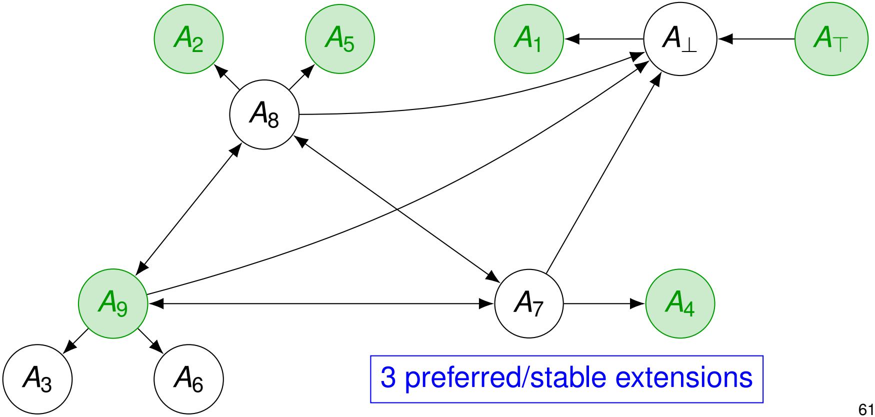

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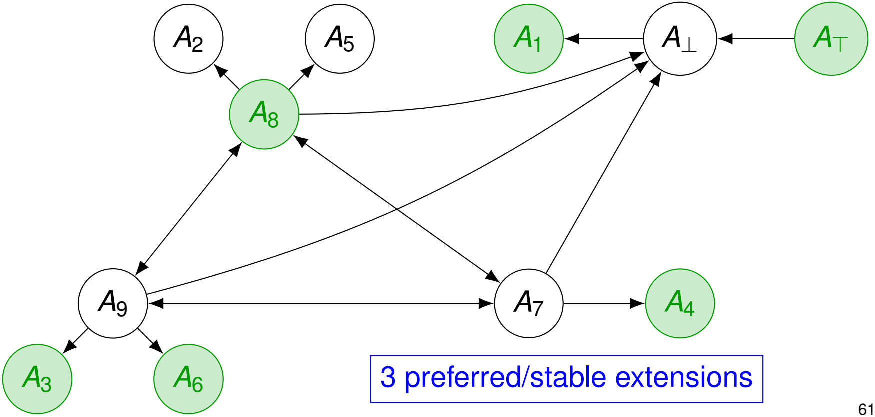

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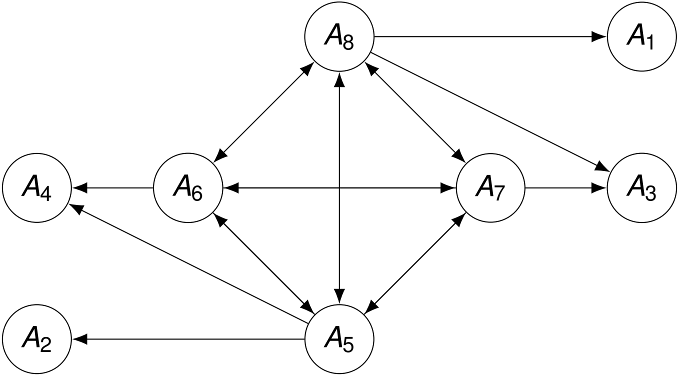

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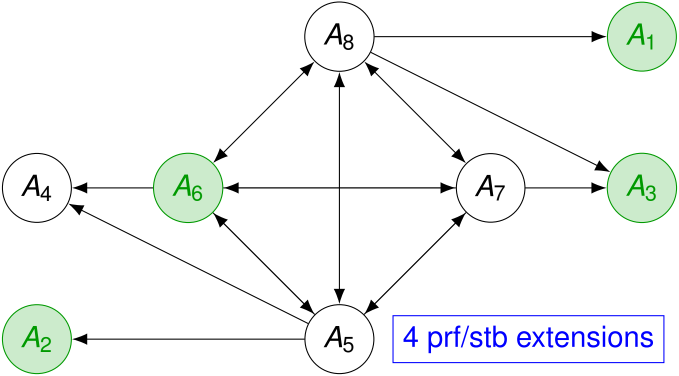

4 prf/stb extensions

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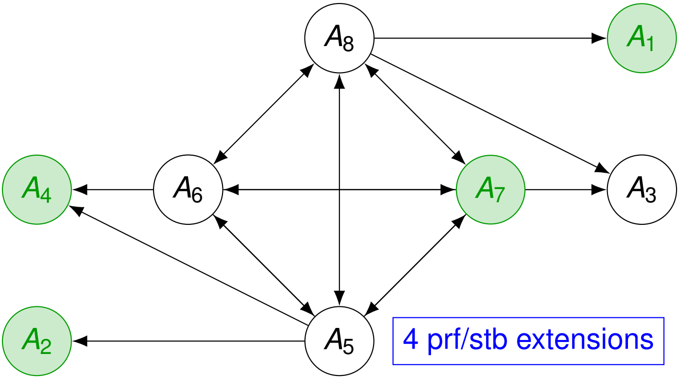

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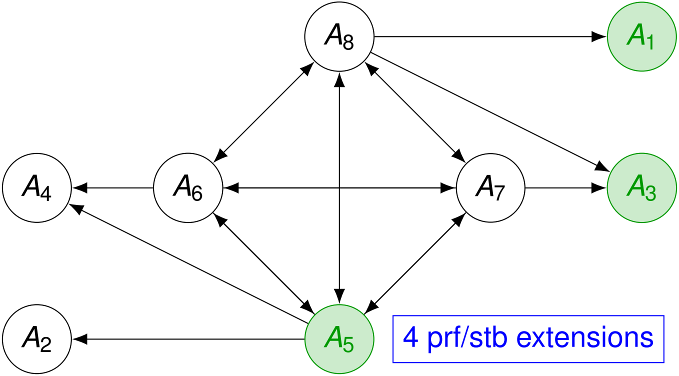

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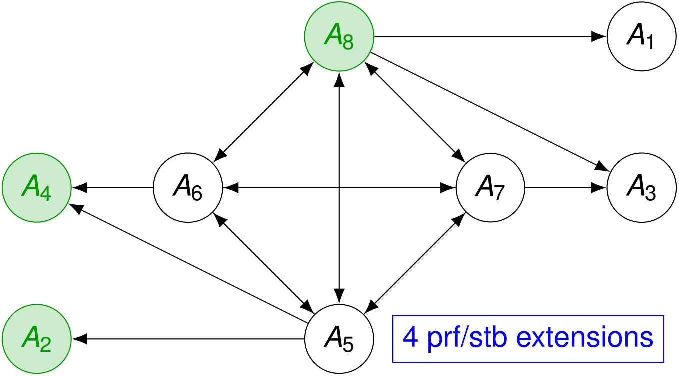

4 prf/stb extensions

Incorporation of Modalities (2)

Example (Horty, 1994)

When a meal is served (m), one should not eat with fingers (f).
However, if the meal is asparagus (a), one should eat with fingers.

J. Horty. *Moral dilemmas and nonmonotonic logic*, Journal of Philosophical Logic, 23:35–65, 1994.

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Representation by a sequent-based AF:

- $\mathfrak{L} = \mathsf{SDL}$ (standard deontic logic, i.e., the normal modal logic KD), where the modal operator O intuitively represents obligations.
- $\mathcal{S} = \{m,\ a,\ m \supset \mathsf{O}\neg f,\ (m \wedge a) \supset \mathsf{O}f\}$.
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This is a paradigmatic case of *specificity*: a more specific obligation cancels (or overrides) a less specific obligation.

Specificity Undercut

$$\frac{\Gamma, \phi \supset \mathsf{O}\psi \Rightarrow \neg(\phi' \supset \mathsf{O}\psi') \quad \Gamma \vdash \phi \quad \phi \vdash \phi' \quad \psi \vdash \neg\psi' \quad \Gamma', \phi' \supset \mathsf{O}\psi' \Rightarrow \sigma}{\Gamma', \phi' \supset \mathsf{O}\psi' \not\Rightarrow \sigma}$$

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Entailments Induced by Sequent-based AFs

- $\mathcal{AF}(\mathcal{S}) = \langle \text{Arg}_{\mathfrak{L}}(\mathcal{S}), \text{Attack}(\mathcal{A})\rangle$ – A sequent-based AF
- $\text{Sem}(\mathcal{AF}(\mathcal{S}))$ – The Sem-extensions of $\mathcal{AF}(\mathcal{S})$ ($\text{Sem} \in \{\text{Cmp}, \text{Grd}, \text{Prf}, \text{Stb}, \text{SStb}\}$).

- $\mathcal{S} \mid\!\sim^{\cap}_{\mathfrak{L},\mathcal{A},\text{sem}} \psi$ if $\exists A \in \bigcap \text{Sem}(\mathcal{AF}(\mathcal{S}))$ with $\text{Conc}(A) = \psi$
- $\mathcal{S} \mid\!\sim^{\Cap}_{\mathfrak{L},\mathcal{A},\text{sem}} \psi$ if $\forall \mathcal{E} \in \text{Sem}(\mathcal{AF}(\mathcal{S}))\ \exists A \in \mathcal{E}$ with $\text{Conc}(A) = \psi$
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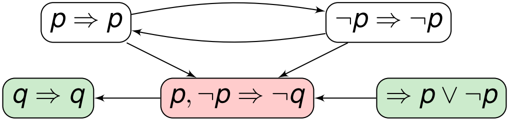

Thus, for every $\star \in \{\cap, \Cap, \cup\}$,
$\{p, \neg p, q\} \mathrel{|\!\sim}^{\star}_{\mathsf{CL},\{\mathsf{Ucut}\},\mathsf{grd}} p \vee \neg p$ and $\{p, \neg p, q\} \mathrel{|\!\sim}^{\star}_{\mathsf{CL},\{\mathsf{Ucut}\},\mathsf{grd}} q$
(but $\{p, \neg p, q\} \not\mathrel{|\!\sim}^{\star}_{\mathsf{CL},\{\mathsf{Ucut}\},\mathsf{grd}} p$ and $\{p, \neg p, q\} \not\mathrel{|\!\sim}^{\star}_{\mathsf{CL},\{\mathsf{Ucut}\},\mathsf{grd}} \neg p$).

Entailments Induced by Sequent-based AFs (Cont'd.)

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$m,\ a,\ m \supset \mathsf{O}\neg f,\ (m \wedge a) \supset \mathsf{O}f\ \mid\!\sim^{\star}_{\mathsf{SDL},\{\mathsf{SpecUcut}\},\mathsf{sem}}\ \mathsf{O}f$

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Notes:

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Example

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- $\mathcal{S} \mid\!\sim^{\cup}_{\mathfrak{L},\mathcal{A},\mathsf{sem}} p$ but $\mathcal{S} \not\mid\!\sim^{\Cap}_{\mathfrak{L},\mathcal{A},\mathsf{sem}} p$, $\mathcal{S} \mid\!\sim^{\Cap}_{\mathfrak{L},\mathcal{A},\mathsf{sem}} q$ but $\mathcal{S} \not\mid\!\sim^{\cap}_{\mathfrak{L},\mathcal{A},\mathsf{sem}} q$.

Entailments Induced by Sequent-based AFs (Cont'd.)

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- Let $\mathcal{S} = \{p \wedge q, \neg p \wedge q\}$, $\mathfrak{L} = \mathsf{CL}$, $\mathcal{A} = \{\mathsf{DirUcut}\}$.
- $\mathsf{Prf}(\mathcal{AF}(\mathcal{S})) = \mathsf{Stb}(\mathcal{AF}(\mathcal{S})) = \{\mathsf{Arg}_{\mathsf{CL}}(p \wedge q)\ ,\ \mathsf{Arg}_{\mathsf{CL}}(\neg p \wedge q)\}$.
- $\mathcal{S} \mid\!\sim^{\cup}_{\mathfrak{L},\mathcal{A},\mathsf{sem}} p$ but $\mathcal{S} \not\mid\!\sim^{\Cap}_{\mathfrak{L},\mathcal{A},\mathsf{sem}} p$, $\mathcal{S} \mid\!\sim^{\Cap}_{\mathfrak{L},\mathcal{A},\mathsf{sem}} q$ but $\mathcal{S} \not\mid\!\sim^{\cap}_{\mathfrak{L},\mathcal{A},\mathsf{sem}} q$.

- Properties of the entailments will be discussed later.

Plan of Module 2

1. Motivation and Introduction
2. Abstract Argumentation Frameworks (AAFs)
   - Basic Definitions, Semantics
   - The Induced Entailments
   - Paraconsistent Semantics
3. Logical (Deductive) Argumentation Frameworks
4. **Some Instantiations**
   - Sequent-based Argumentation
   - **ASPIC Systems**
   - Assumption-based Argumentation

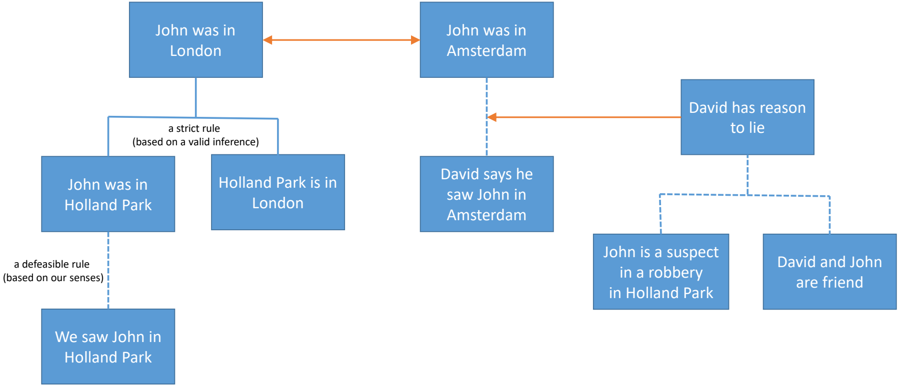
ASPIC Systems (Informal Presentation)

Some Basic Principles

- Argumentation based on deductive arguments (similar to sequent-based argumentation).
- The background knowledge consists of strict (non-attackable) assumptions and defeasible (attackable) assumptions.
- Derivations are based on strict (deductively valid) and defeasible (presumptive) rules.
- Arguments are of the form $\langle \Gamma, \psi \rangle$, where the support Γ is a tree-structured derivation of ψ (using the available rules).
- Attacks on the defeasible rules in the support.
- Standard Dung-style semantics on the induced AF.

S. Modgil, H. Prakken, *Abstract rule-based argumentation*, Handbook of Formal Argumentation, Vol.I, pp.287–364, 2018.

ASPIC Systems – Intuition and Motivation



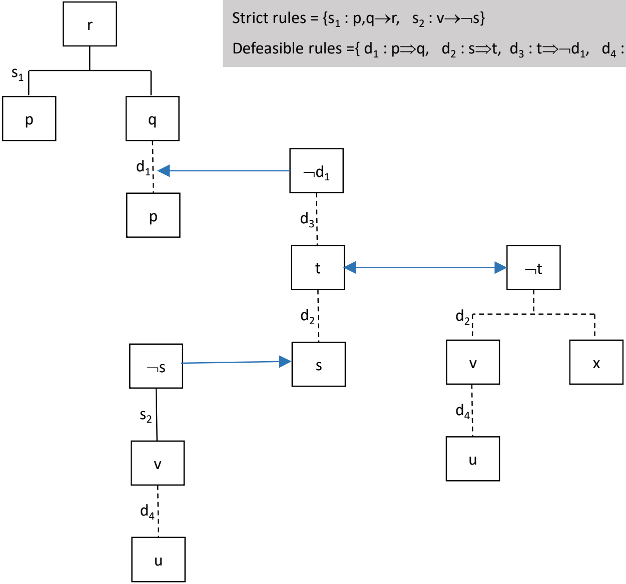
(Taken from S. Modgil amd H. Prakken, “Abstract rule-based argumentation”, Handbook of Formal Argumentation Vol.I:287--364, 2018)

ASPIC Systems – A More Detailed Example

Strict premises = {p} , Defeasible premises = {s,u,x}

Strict rules = {s_1 : p,q→r, s_2 : v→¬s}

Defeasible rules ={ d_1 : p⇒q, d_2 : s⇒t, d_3 : t⇒¬d_1, d_4 : u⇒v, d_5 : v,x⇒¬t }



ASPIC Systems – A More Detailed Example (Cont'd.)

Strict premises = {p} , Defeasible premises = {s,u,x}

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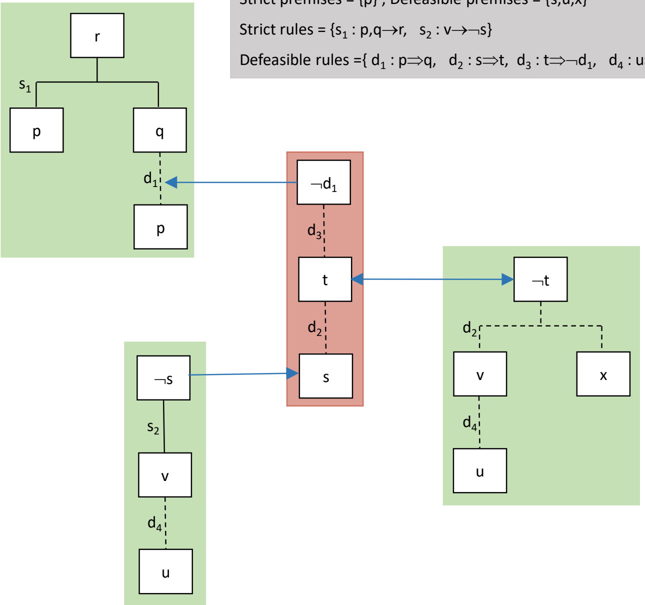

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Assumption-Based Argumentation (ABA)

Basic Idea: ABA systems operate on sets of *assumptions* (formulas) rather than individual arguments. This may be viewed as a higher level of abstraction, operating on equivalence classes that consist of arguments generated from the same assumptions.

K. Cyras, X. Fan, C. Schulz., F. Toni. *Assumption-based argumentation: Disputes, explanations, preferences*, Handbook of Formal Argumentation, Vol.I, pp.365–408, 2018.

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An *assumption-based framework* is a tuple $\mathcal{ABF} = \langle \mathcal{L}, \Gamma, \Delta, \sim \rangle$, s.t.:

- $\mathcal{L}$ is a (propositional) language,
- Γ is a set of *strict rules* of the form $\psi_1, \ldots, \psi_n \rightarrow \psi$,
- Δ is a set of $\mathcal{L}$-formulas, called the *defeasible assumptions*,
- $\sim : \Delta \rightarrow 2^{\mathcal{L}}$ is a *contrariness operator*.

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$\mathcal{S} \vdash \psi$ if there is a Γ-deduction based on $\mathcal{S} \subseteq \Delta$ that culminates in ψ.

$\mathcal{S}$ *attacks* ψ if there are $\mathcal{S}' \subseteq \mathcal{S}$ and $\phi \in {\sim}\psi$ such that $\mathcal{S}' \vdash \phi$.

$\mathcal{S} \subseteq \Delta$ attacks $\mathcal{T} \subseteq \Delta$ if $\mathcal{S}$ attacks some $\psi \in \mathcal{T}$.

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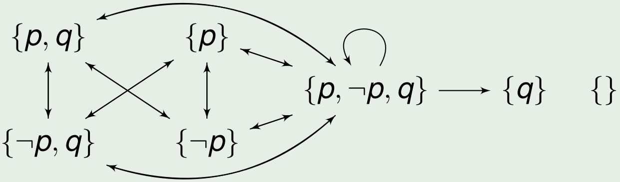
Assumption-Based Argumentation, Cont'd.

Example

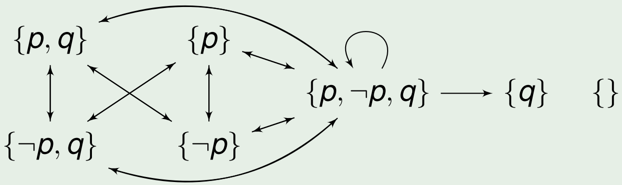


Attack diagram for $\langle \mathcal{L}, \Gamma, \Delta, \sim \rangle$

$\Delta = \{p, \neg p, q\}, \; \Gamma = \{x \to x; \;\; p, \neg p \to x \mid x \in \Delta\}, \; \sim\psi = \{\neg\psi\}$

Assumption-Based Argumentation, Cont'd.

Example



Attack diagram for $\langle \mathcal{L}, \Gamma, \Delta, \sim \rangle$

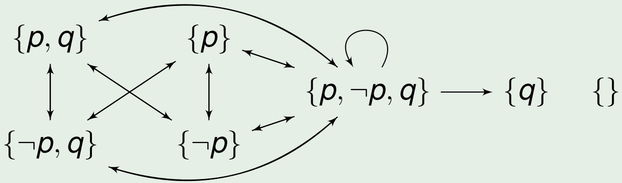
$\Delta = \{p, \neg p, q\},\ \Gamma = \{x \to x;\ \ p, \neg p \to x \mid x \in \Delta\},\ \sim\psi = \{\neg\psi\}$

Let $\mathcal{ABF} = \langle \mathcal{L}, \Gamma, \Delta, \sim \rangle$ and $\mathcal{S} \subseteq \Delta$

- $\mathcal{S}$ is *Δ-closed* if: $\mathcal{S} = \Delta \cap \{\psi \mid \mathcal{S} \vdash \psi\}$
- $\mathcal{S}$ is *conflict-free* iff it does not attack itself.
- $\mathcal{S}$ *defends* a set $\mathcal{S}' \subseteq \Delta$ iff for every closed set $\mathcal{S}''$ that attacks $\mathcal{S}'$, $\mathcal{S}$ attacks $\mathcal{S}''$.
- $\mathcal{S}$ is *admissible* iff it is closed, conflict-free, and defends itself. An admissible set is *complete* if it does not defend any of its proper supersets (grd, prf, stb, sstb, etc. are defined as usual).

Assumption-Based Argumentation, Cont'd.

Example



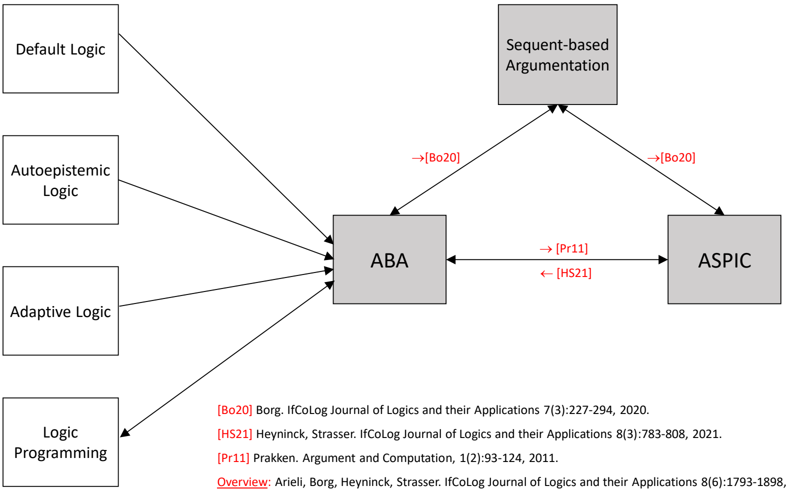
Attack diagram for $\langle \mathcal{L}, \Gamma, \Delta, \sim \rangle$

$\Delta = \{p, \neg p, q\}$, $\Gamma = \{x \rightarrow x;\ \ p, \neg p \rightarrow x \mid x \in \Delta\}$, $\sim\psi = \{\neg\psi\}$

- $\mathsf{Prf}(\mathcal{ABF}) = \mathsf{Stb}(\mathcal{ABF}) = \{\{p, q\}, \{\neg p, q\}\}$
- $p, \neg p, q \not\mid\!\sim^{\star}_{\mathsf{sem}} p$ and $p, \neg p, q \not\mid\!\sim^{\star}_{\mathsf{sem}} \neg p$, while $p, \neg p, q \mid\!\sim^{\star}_{\mathsf{sem}} q$ for every $\star \in \{\cup, \cap, \Cap\}$ and sem $\in \{\mathsf{Prf}, \mathsf{Stb}\}$.

Some Relations Among The Formalisms

(To be discussed also in Module 5)



[Bo20] Borg. IfCoLog Journal of Logics and their Applications 7(3):227-294, 2020.

[HS21] Heyninck, Strasser. IfCoLog Journal of Logics and their Applications 8(3):783-808, 2021.

[Pr11] Prakken. Argument and Computation, 1(2):93-124, 2011.

Overview: Arieli, Borg, Heyninck, Strasser. IfCoLog Journal of Logics and their Applications 8(6):1793-1898, 2021.