Reasoning with Modularly Pointwise Preferential Relations

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Abstract

We introduce a family of preferential consequence relations, defined by a simple and natural many-valued semantics. These relations share many desirable properties for common-sense reasoning, such as paraconsistency (da-Costa, [8]), plausibility (Lehmann, [12]), adaptivity (Batens, [4, 5]), and rationality (Lehmann and Magidor, [13]).

1 Introduction

Preferential reasoning [21] is a well-known formalism for making inferences, based on the idea that in order to draw conclusions from a given theory one should not consider all the models of that theory, but only a subset of *preferred models*. This subset is usually determined according to some preference criterion, specified by a second-order formula (as in circumscription [16]), or by some (partial) order that is defined on the valuation space [1, 2, 17, 18]. This approach is particularly useful for providing semantics to general patterns of nonmonotonic reasoning [3, 11, 13, 14, 15].

In this paper we consider a family of preferential consequence relations, defined by a general and natural semantics. The common property shared by all these relations is that their underlying preference criteria are based on *modular partial orders*. We show that this property enables a "robust" construction of consequence relations, in the sense that such relations may be plausibility logics [12] with adaptive capabilities [4, 5]. Moreover, many paraconsistent [8] consequence relations that are definable within our framework are the same as classical logic w.r.t. consistent theories. This allows us to consider formalisms that draw classical conclusions from consistent theories, and make non-trivial conclusions from inconsistent ones.

2 Preliminaries

Definition 1 A partial order $<$ on a set S is called *modular* if $y < x_2$ for every $x_1, x_2, y \in S$ s.t. $x_1 \not< x_2$, $x_2 \not< x_1$, and $y < x_1$.

Proposition 2 [13] Let $<$ be a partial order on S. The following conditions are equivalent:

a) $<$ is modular.

b) If $x_1 < x_2$ then either $y < x_2$ or $x_1 < y$ for every $x_1, x_2, y \in S$.

c) There is a totally ordered set S' with a strict order $\prec$ and a function $g : S \to S'$ s.t. $x_1 < x_2$ iff $g(x_1) \prec g(x_2)$.

Definition 3 Let Σ be an arbitrary propositional language. A *preferential structure* for Σ is a quadruple $\mathcal{P} = (\mathcal{L}, \mathcal{D}, \mathcal{O}, \leq)$, where $\mathcal{L}$ is a complete bounded lattice, $\mathcal{D} \subset \mathcal{L}$ is a prime filter in $\mathcal{L}$ that contains the *designated* elements of $\mathcal{L}$,[1] $\mathcal{O}$ is a set of operations on $\mathcal{L}$ that correspond to the connectives in Σ, and $\leq$ is a well-founded modular order on $\mathcal{L}$.

In what follows we shall denote the maximal element of $\mathcal{L}$ by t and the minimal one by f. We shall assume that the set $\mathcal{O}$ contains a meet and a join operations that correspond, respectively, to the conjunction ($\wedge$) and the disjunction ($\vee$) in Σ, and an involution operation that corresponds to the negation operator ($\neg$) in Σ.

The semantical notions that correspond to the multiple-valued case are natural generalizations of the classical ones: A (multiple-valued) *valuation* ν is a function that assigns an element of $\mathcal{L}$ to each atomic formula. Extension to complex formulae is done in the standard way. A valuation ν is a *model* of a set Γ of assertions if $\nu(\psi) \in \mathcal{D}$ for every $\psi \in \Gamma$. The set of all the valuations into $\mathcal{L}$ is denoted by $\mathcal{V}^{\mathcal{L}}$, and the set of all the models of Γ is denoted by $mod(\Gamma)$.

Definition 4 Let $\nu_1, \nu_2 \in \mathcal{V}^{\mathcal{L}}$, and let $<$ be a modular order on $\mathcal{L}$. Denote:

a) $\nu_1 \preceq \nu_2$ if for every atom p $\nu_2(p) \not< \nu_1(p)$.

b) $\nu_1 \prec \nu_2$ if $\nu_1 \preceq \nu_2$ and there is an atom p_0 s.t. $\nu_1(p_0) < \nu_2(p_0)$.[2]

Definition 5 Let $\mathcal{P}$ be a preferential structure, and let Γ be a set of formulae in a language Σ. A valuation $M \in mod(\Gamma)$ is a *$\mathcal{P}$-preferential model* of Γ if there is no other valuation $M' \in mod(\Gamma)$ s.t. $M' \prec M$. The set of all the $\mathcal{P}$-preferential models of Γ is denoted by $!(\Gamma, \mathcal{P})$.

Definition 6 Let $\mathcal{P} = (\mathcal{L}, \mathcal{D}, \mathcal{O}, \leq)$ be a preferential structure. A set Γ of formulae *$\mathcal{P}$-preferentially entails* a set Δ of formulae (notation: $\Gamma \models_{\leq}^{\mathcal{L},\mathcal{D}} \Delta$) if every $M \in\ !(\Gamma, \mathcal{P})$ is a model of some formula in Δ.[3]

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[1] I.e., those elements that represent true assertions.

[2] Note that $\preceq$ is a pre-order and $\prec$ is a strict order (i.e., irreflexive and transitive).

[3] In what follows will we say that $\models_{\leq}^{\mathcal{L},\mathcal{D}}$ is *induced* by $\mathcal{P}$.

3 Preferential consequence relations

Many well-known formalisms correspond to Definition 6. In this section we consider some of them.

Suppose first that $\mathcal{P}$ is a preferential structure with a degenerated preferential order. I.e., all the elements in $\mathcal{L}$ are $\leq$-incomparable. In this case Γ $\mathcal{P}$-preferentially entails Δ if every model of Γ is a model of some formula in Δ. In case that $\mathcal{L}$ is a two-valued lattice the consequence relation that is obtained is that of classical logic. Kleene three-valued logic [10] obtains by taking a three-valued lattice $\mathcal{L}=\{t,f,\bot\}$, where $\bot$ is the middle element, and $\mathcal{D}=\{t\}$. Belnap four-valued logic [6, 7] obtains by taking a four-valued lattice $\mathcal{L}=\{t,f,\top,\bot\}$, in which $\bot$ and $\top$ are two intermediate incomparable elements, and $\mathcal{D}=\{t,\top\}$. For arbitrary preferential structures with degenerated preferential orders, we have:

Proposition 7 [3] A relation $\models^{\mathcal{L},\mathcal{D}}_{\leq}$, induced by a preferential structure with a degenerated preferential order, is a consequence relation in the sense of Tarski [23] and Scott [20]. I.e., it satisfies the following conditions:

reflexivity:	if $\Gamma\cap\Delta\neq\emptyset$ then $\Gamma\models^{\mathcal{L},\mathcal{D}}_{\leq}\Delta$.
monotonicity:	if $\Gamma\models^{\mathcal{L},\mathcal{D}}_{\leq}\Delta$ and $\Gamma\subseteq\Gamma'$, $\Delta\subseteq\Delta'$, then $\Gamma'\models^{\mathcal{L},\mathcal{D}}_{\leq}\Delta'$.
cut:	if $\Gamma_1\models^{\mathcal{L},\mathcal{D}}_{\leq}\psi,\Delta_1$ and $\Gamma_2,\psi\models^{\mathcal{L},\mathcal{D}}_{\leq}\Delta_2$ then $\Gamma_1,\Gamma_2\models^{\mathcal{L},\mathcal{D}}_{\leq}\Delta_1,\Delta_2$.

In the general case, when the pointwise preferential order is *not* degenerated, $\models^{\mathcal{L},\mathcal{D}}_{\leq}$ is usually non-monotonic (see Section 4.1). Below are some formalisms that are obtained in this case:

• *Reiter's closed-world assumption* [19]: Obtains by using a two-valued lattice and a pointwise preferential order that is the same as the partial order of the lattice under consideration (which is clearly modular). The preferential models of a theory in this case are those that minimize the amount of the t-assignments. In case of first-order languages the preferential models of a theory are its minimal Herbrand models.

• *The logic LPm of Priest* [17, 18]: This logic is based on a three-valued lattice $\mathcal{L}=\{t,f,\top\}$ with a middle element $\top$, and $\mathcal{D}=\{t,\top\}$. Here $\neg\top=\top\in\mathcal{D}$, and so $\top$ is intuitively understood as representing contradictions. The preferred models in this case are those that are minimally inconsistent, i.e.: those that assign $\top$ only to some minimal set of atomic formulae. The modular preferential order may therefore be defined here by: $f<\top$ and $t<\top$.

• *The four-valued logic* $\models^4_k$: The preferred models in this case are those that are minimal with respect to the "knowledge order" $\leq_k$ of Belnap's four-valued information lattice $A4$ [6, 7]. In this (modularly) preferential order $\bot$ is the minimal element, t and f are two intermediate elements that are $\leq_k$-incomparable, and $\top$ is the maximal element.

• *The logics* $\models^4_{\mathcal{I}_1}$ *and* $\models^4_{\mathcal{I}_2}$ [1, 2]: Again, these logics are based on Belnap's four-valued lattice [6, 7]. The preferential order for $\models^4_{\mathcal{I}_1}$ obtains by taking $\top$ as the only maximal element (and all the other truth values are $\leq$-incomparable). The preferential order for $\models^{4}_{\mathcal{I}_2}$ obtains by taking $\top$ and $\bot$ as $\leq$-greater than t and f (see [2] for a justification of these choices).

• *The logic RI of Kifer and Lozinskii* [9]: This is an annotated logic [22]. The preferred models of RI minimize the assignments w.r.t. a certain set $\Delta \subset \mathcal{L}$. Thus, the preferential order in this case obtains by considering every element in Δ as strictly $\leq$-greater than every element in $\mathcal{L} \setminus \Delta$.

4 Useful properties of $\models^{\mathcal{L},\mathcal{D}}_{\leq}$

4.1 Non-monotonicity and plausibility

In Proposition 7 we have shown that $\models^{\mathcal{L},\mathcal{D}}_{\leq}$ is monotonic in cases that the preferential order under consideration is degenerated. In the general case, however, relations of the form $\models^{\mathcal{L},\mathcal{D}}_{\leq}$ are non-monotonic.[4] In such cases it is usual to require weaker conditions, such as those introduced by Kraus, Lehmann, and Magidor in the context of *preferential logics* [11, 14, 15], or those of Lehmann's *plausibility logics* [12].

Definition 8 [12] A binary relation $\mid\!\sim$ between sets of formulae is called a *plausibility logic* if the following properties are satisfied:

Inclusion:	$\Gamma, \psi \mid\!\sim \psi$.
Right Monotonicity:	If $\Gamma \mid\!\sim \Delta$, then $\Gamma \mid\!\sim \psi, \Delta$.
Cautious Left Monotonicity:	If $\Gamma \mid\!\sim \psi$ and $\Gamma \mid\!\sim \Delta$, then $\Gamma, \psi \mid\!\sim \Delta$.
Cautious Cut:	If $\Gamma, \psi \mid\!\sim \Delta$ and $\Gamma \mid\!\sim \psi, \Delta$, then $\Gamma \mid\!\sim \Delta$.

Definition 9 A preferential structure $\mathcal{P}$ is called *stoppered* [15] [5] if for every set of formulae Γ and every $M \in mod(\Gamma)$, either $M \in\, !(\Gamma, \mathcal{P})$, or there is an $M' \in\, !(\Gamma, \mathcal{P})$ s.t. $M' \prec M$.

Proposition 10 Let $\mathcal{P} = (\mathcal{L}, \mathcal{D}, \mathcal{O}, \leq)$ be a stoppered preferential structure. Then $\models^{\mathcal{L},\mathcal{D}}_{\leq}$ is a plausibility logic.

Proof: Inclusion and Right Monotonicity immediately follow from the definition of $\models^{\mathcal{L},\mathcal{D}}_{\leq}$. For Cautious Left Monotonicity, assume that $\Gamma \models^{\mathcal{L},\mathcal{D}}_{\leq} \psi$, and $\Gamma \models^{\mathcal{L},\mathcal{D}}_{\leq} \Delta$. Let M be some $\mathcal{P}$-preferential model of $\Gamma \cup \{\psi\}$. In particular, M is a model of Γ. Moreover, it must be a $\mathcal{P}$-preferential model of Γ as well, since otherwise, by stopperdness, there would have been an $N \in\, !(\Gamma, \mathcal{P})$ s.t. $N \prec M$. Since $\Gamma \models^{\mathcal{L},\mathcal{D}}_{\leq} \psi$, this N would have been a model of $\Gamma \cup \{\psi\}$ that is strictly $\prec$-smaller than M. Hence $M \notin\, !(\Gamma \cup \{\psi\}, \mathcal{P})$, with a contradiction to the choice of M. Thus, $M \in\, !(\Gamma, \mathcal{P})$. Now, since $\Gamma \models^{\mathcal{L},\mathcal{D}}_{\leq} \Delta$, M is a model of some $\delta \in \Delta$. Hence $\Gamma, \psi \models^{\mathcal{L},\mathcal{D}}_{\leq} \Delta$.

It remains to show Cautious Cut. For this, let M be a $\mathcal{P}$-preferential model of Γ. Suppose, for a contradiction, that $\Gamma, \psi \models^{\mathcal{L},\mathcal{D}}_{\leq} \Delta$ and $\Gamma \models^{\mathcal{L},\mathcal{D}}_{\leq} \psi, \Delta$, but $M(\delta) \notin \mathcal{D}$

[4] For instance, all the logics considered after Proposition 7 are nonmonotonic.

[5] In [11] the same property is called *smoothness*.

for every $\delta \in \Delta$. Since $\Gamma \models_{\leq}^{\mathcal{L},\mathcal{D}} \psi, \Delta$, necessarily $M(\psi) \in \mathcal{D}$, and so M is a model of $\Gamma \cup \{\psi\}$. Moreover, M must be a $\mathcal{P}$-preferential model of $\Gamma \cup \{\psi\}$, since any other model of this set that is strictly $\prec$-smaller than M must be in particular a model of Γ, which is $\prec$-smaller than M (and this contradicts the choice of M). Now, $\Gamma, \psi \models_{\leq}^{\mathcal{L},\mathcal{D}} \Delta$, therefore $M(\delta) \in \mathcal{D}$ for some $\delta \in \Delta$, a contradiction. □

Corollary 11 If $\mathcal{L}$ is finite, then $\models_{\leq}^{\mathcal{L},\mathcal{D}}$ is a plausibility logic w.r.t. finite sets of premises.

Proof: Follows from Proposition 10, since the conditions of the corollary yield stopperdness. □

In the next proposition (the proof of which will be given elsewhere) it is shown that consequence relations of the form $\models_{\leq}^{\mathcal{L},\mathcal{D}}$ may also be considered as a generalization to the multiple-valued case of the 2-valued patterns of nonmonotonic reasoning, considered in [11].

Proposition 12 If $\mathcal{L}$ is a two valued lattice, and the language Σ is the classical propositional one, then the single-assumption-single-conclusion fragment of $\models_{\leq}^{\mathcal{L},\mathcal{D}}$ is a preferential logic in the sense of Kraus, Lehmann, and Magidor [11].

4.2 Paraconsistency and classicality

A desirable property of formalisms for managing inconsistent information is that they will be able to draw classical conclusions from (classically) consistent theories, and will not "explode" the set of conclusions when the theory becomes inconsistent. Corollary 16 shows that many consequence relations that are induced by preferential structures have this property.

Definition 13 A modular order $\leq$ on $\mathcal{L}$ is called *classical* if t and f are the only (incomparable) $\leq$-minimal elements.

In what follows we denote by $\models^2$ the classical consequence relation.

Proposition 14 Let $\mathcal{P} = (\mathcal{L}, \mathcal{D}, \mathcal{O}, \leq)$ be a pointwise preferential structure, where $\leq$ is a classical modular order on $\mathcal{L}$. For every classically consistent theory Γ, and for every formula ψ, we have that $\Gamma \models^2 \psi$ iff $\Gamma \models_{\leq}^{\mathcal{L},\mathcal{D}} \psi$.

Proof: Immediately follows from the fact that if $\leq$ is a classical modular order and Γ is a classically consistent theory, then $!(\Gamma, \mathcal{P})$ coincides with the set of the classical models of Γ. □

Proposition 15 If there is an element $x \in \mathcal{L}$ s.t. $x, \neg x \in \mathcal{D}$ and there is a $\leq$-minimal element y s.t. either $y \notin \mathcal{D}$ or $\neg y \notin \mathcal{D}$, then $\models_{\leq}^{\mathcal{L},\mathcal{D}}$ is paraconsistent.

Proof: Consider, for instance, $\Gamma = \{p, \neg p\}$. Although Γ is classically inconsistent, not every conclusion follows from it. In fact, for every atom $q \neq p$ we have $\Gamma \not\models_{\leq}^{\mathcal{L},\mathcal{D}} q$. To see that, consider a valuation M, for which $M(p) = x$ and $M(q) = y$ for $y \notin \mathcal{D}$. □

By Propositions 14 and 15 we have the following result:

Corollary 16 Let $\mathcal{P} = (\mathcal{L}, \mathcal{D}, \mathcal{O}, \leq)$ be a pointwise preferential structure, where there is an $x \in \mathcal{L}$ s.t. $x, \neg x \in \mathcal{D}$, and $\leq$ is a classical modular order on $\mathcal{L}$ for which there is a $\leq$-minimal element y s.t. either $y \notin \mathcal{D}$ or $\neg y \notin \mathcal{D}$. Then $\models_{\leq}^{\mathcal{L},\mathcal{D}}$ is the same as the classical consequence relation w.r.t. consistent theories, and is not trivial w.r.t. inconsistent theories.

The consequence relation of LPm and the consequence relation $\models_{\mathcal{I}_2}^{4}$, considered in Section 3, are examples of formalisms that have the property specified in the last corollary.

4.3 Rationality

In [13] Lehmann and Magidor consider some properties that a "rational" non-monotonic consequence relation should satisfy. One property that is considered as particularly important assures that a reasoner will not have to retract any previous conclusion when learning about a new fact that has no influence on the existing set of premises.[6] Consequence relations that satisfy this property are called *rational*. Next we show that the relations that are induced by preferential structures are indeed "rational".

Notation 17 Denote by $\mathcal{A}(\Gamma)$ the set of the atomic formulae that appear in some formula of Γ.

Proposition 18 If $\Gamma \models_{\leq}^{\mathcal{L},\mathcal{D}} \Delta$ and $\mathcal{A}(\Gamma \cup \Delta) \cap \mathcal{A}(\Lambda) = \emptyset$, then $\Gamma, \Lambda \models_{\leq}^{\mathcal{L},\mathcal{D}} \Delta$.

Proof: Suppose otherwise that $\Gamma, \Lambda \not\models_{\leq}^{\mathcal{L},\mathcal{D}} \Delta$. Then there is a model $M \in\, !(\Gamma \cup \Lambda, \mathcal{P})$ such that for every $\delta \in \Delta$, $M(\delta) \notin \mathcal{D}$. Let m be some $\leq$-minimal element in $\mathcal{L}$. Consider the following valuation:

$$N(p) = \begin{cases} M(p) & \text{if } p \in \mathcal{A}(\Gamma \cup \Delta) \\ m & \text{otherwise} \end{cases}$$

Clearly, N is a model of Γ and for every $\delta \in \Delta$, $N(\delta) \notin \mathcal{D}$. Since $\Gamma \models_{\leq}^{\mathcal{L},\mathcal{D}} \Delta$, N cannot be a $\mathcal{P}$-preferential model of Γ, and so there is a model N' of Γ s.t. $N' \prec N$. By the definition of N, there is some $p_0 \in \mathcal{A}(\Gamma \cup \Delta)$ such that $N'(p_0) < N(p_0)$. Now, consider the following valuation:

$$M'(p) = \begin{cases} N'(p) & \text{if } p \in \mathcal{A}(\Gamma \cup \Delta) \\ M(p) & \text{otherwise} \end{cases}$$

Clearly, $M' \prec M$, and since M' is the same as N' on $\mathcal{A}(\Gamma)$, M' is also a model of Γ. Moreover, using the facts that $\mathcal{A}(\Gamma \cup \Delta) \cap \mathcal{A}(\Lambda) = \emptyset$ and that M is a model of Λ, it follows that M' is also a model of Λ. Hence M' is a model of $\Gamma \cup \Lambda$, which is strictly $\prec$-smaller than M, but this is a contradiction to the choice of M. □

[6] E.g., using the example considered in [11], the fact that a certain bird is red should not affect our knowledge about its flying abilities.

4.4 Adaptivity

Consider the set $\Gamma_1 = \{p, \neg p, \neg p \vee q\}$. A plausible inference mechanism should *not* apply the Disjunctive Syllogism here to p and $\neg p \vee q$ for concluding that q follows from Γ_1. The reason for this is that $\neg p$ holds in Γ_1 and so $\neg p \vee q$ is true even in cases that q is false. On the other hand, in the case of $\Gamma_2 = \{p, \neg p, r, \neg r \vee q\}$, applying the Disjunctive Syllogism to r and $\neg r \vee q$ may be justified by the fact that the subset of formulae to which the Disjunctive Syllogism is applied should not be affected by the inconsistency in Γ_2, therefore inference rules that are classically valid can be applied to it.

The ability to handle theories with contradictions in a nontrivial way, but presuppose a consistency of all sentences 'unless and until proven otherwise', is called *adaptivity* [4, 5]. Consequence relations with this property *adapt* to the *specific* inconsistencies that occur in the theories.

The following proposition shows that preferential relations that are based on classical modular orders are adaptive: If a given theory can be split up to a consistent and an inconsistent parts, then every assertion that is not related to the inconsistent part, and which classically follows from the consistent part, must preferentially follow from of the whole theory.

Proposition 19 Let $(\mathcal{L}, \mathcal{D}, \mathcal{O}, \leq)$ be a pointwise preferential structure, where $\leq$ is a classical modular order on $\mathcal{L}$. Let $\Gamma = \Gamma' \cup \Gamma''$ s.t. Γ' is classically consistent and $\mathcal{A}(\Gamma') \cap \mathcal{A}(\Gamma'') = \emptyset$. For every set Δ s.t. $\mathcal{A}(\Delta) \cap \mathcal{A}(\Gamma'') = \emptyset$, if $\Gamma' \models^2 \Delta$ then $\Gamma \models_{\leq}^{\mathcal{L},\mathcal{D}} \Delta$.

Proof: Suppose that $\Gamma' \models^2 \Delta$. By Proposition 14, $\Gamma' \models_{\leq}^{\mathcal{L},\mathcal{D}} \Delta$. But we have here that $\mathcal{A}(\Gamma' \cup \Delta) \cap \mathcal{A}(\Gamma'') = \emptyset$, thus, by Proposition 18, $\Gamma \models_{\leq}^{\mathcal{L},\mathcal{D}} \Delta$. □

5 Conclusion

We have considered a uniform way of defining different consequence relations for commonsense reasoning. It is shown that the consequence relations that are obtained in this way have several useful properties that are important for applications of logic in AI, where uncertainty, inconsistency, and nonmonotonicity have a central role. Such cases are considered, e.g., in [2, 9, 13, 19]. Further applications will be considered in a future work.

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